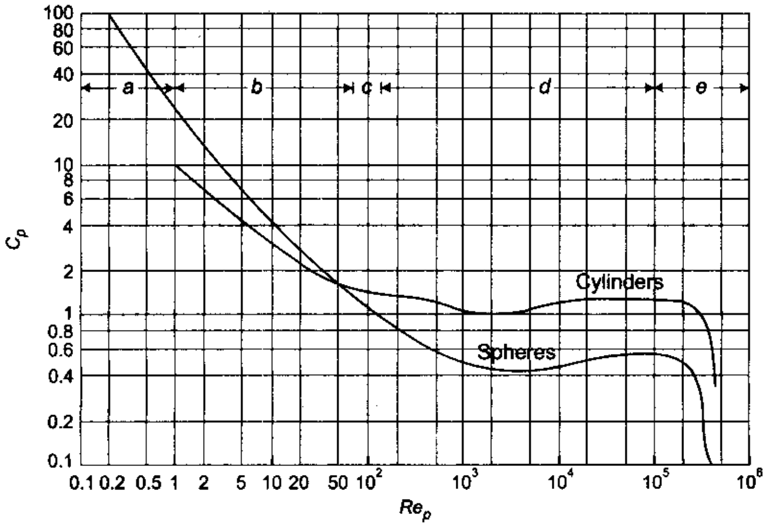


FIGURE 9.15 Drag Coefficient Versus Reynolds Number for Long Circular Cylinders and Spheres in Cross-Flow

Average drag coefficient C_D for cross flow over a cylinder and sphere are shown in Fig. 9.15. Then, the drag force acting on the body in cross flow is obtained from:

$$F_D = C_D \cdot A_N \cdot \frac{\rho \cdot U^2}{2}, \text{ N}$$

where A_N is the 'frontal area' i.e. area normal to the direction of flow.

$$A_N = L.D \qquad \text{...for a cylinder of length } L$$

and,

$$A_N = \frac{\pi \cdot D^2}{4} \qquad \text{...for a sphere}$$

In Fig. 9.15, there are 5 sections, a, b, c, d and e shown. Comments corresponding to these sections of the figure are given below:

(a) At *Re* < 1, inertia forces are negligible and the flow adheres to the surface and drag is only by viscous forces. Heat transfer is purely by conduction.

(b) At *Re* = about 10, inertia forces become appreciable; now, pressure drag is about half of the total drag.

(c) At *Re* of the order of 100, vortices separate and the pressure drag predominates.

(d) At *Re* values between about 1000 and 100,000, skin friction drag is negligible compared to the pressure drag. Point of separation is at about θ = 80 deg. measured from the stagnation point.

(e) At *Re* > 100,000, flow in the boundary layer becomes turbulent and the separation point moves to the rear.

Heat transfer coefficient: Because of the complex nature of flow, most of the results are empirical relations derived from experiments.

Variation of local Nusselt number around the periphery of a cylinder in cross flow is given in Fig. 9.16. *Nu* is high to start with at the stagnation point, then decreases as θ increases due to the thickening of laminar boundary layer. For the two curves at the bottom, minimum is reached at about θ = 80 deg., the separation point in laminar flow. For the rest of the curves, there is a sharp increase at about θ = 90 deg. due to transition from laminar to turbulent flow; *Nu* reaches a second minimum at about θ = 140 deg. due to flow separation in turbulent flow, and thereafter increases with θ, due to intense mixing in the turbulent wake region.

Between θ = 0 and 80 deg. empirical equation for local heat transfer coefficient is:

$$Nu(\theta) = \frac{h_c(\theta) \cdot D}{k} = 1.14 \cdot \left(\frac{\rho \cdot U \cdot D}{\mu}\right)^{0.5} \cdot Pr^{0.4} \cdot \left[1 - \left(\frac{\theta}{90}\right)^3\right] \qquad \text{...(9.89)}$$

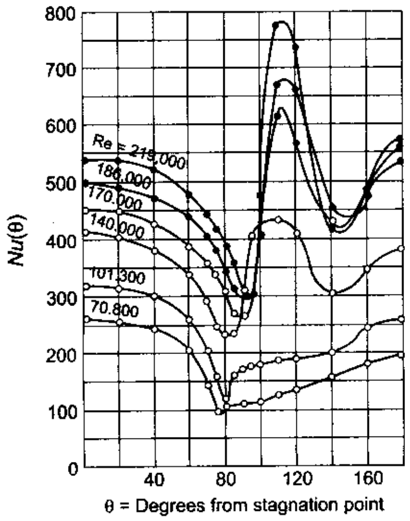


FIGURE 9.16 Circumferential Variation of the Heat Transfer Coefficient at High Reynolds Numbers for a Circular Cylinder in Cross Flow (W.H. Giedt)

While calculating heat transfer coefficient for a cylinder in cross flow, of practical interest is the average heat transfer coefficient over the entire surface. A comprehensive relation for cross flow across a cylinder is given by Churchill and Bernstein:

$$Nu_{cyl} = \frac{h \cdot D}{k} = 0.3 + \frac{0.62 \cdot Re^{\frac{1}{2}} \cdot Pr^{\frac{1}{3}}}{\left[1 + \left(\frac{0.4}{Pr}\right)^{\frac{2}{3}}\right]^{\frac{1}{4}}} \cdot \left[1 + \left(\frac{Re}{28200}\right)^{\frac{5}{8}}\right]^{\frac{4}{5}} \qquad \text{...(9.90)}$$

Eq. 9.90 is valid for $100 < Re < 10^7$, and $Re.Pr > 0.2$ and correlates very well all available data. Fluid properties are evaluated at 'film temperature', $T_f = (T_s + T_a)/2$ = average of surface and free stream temperatures.

In the mid-range of Reynolds numbers, i.e. $20{,}000 < Re < 400{,}000$, it is suggested that following equation be used:

$$Nu_{cyl} = \frac{h \cdot D}{k} = 0.3 + \frac{0.62 \cdot Re^{\frac{1}{2}} \cdot Pr^{\frac{1}{3}}}{\left[1 + \left(\frac{0.4}{Pr}\right)^{\frac{2}{3}}\right]^{\frac{1}{4}}} \cdot \left(1 + \left(\frac{Re}{28200}\right)^{\frac{1}{2}}\right) \qquad \text{...(9.91)}$$

for $20{,}000 < Re < 400{,}000$, and $Re \cdot Pr > 0.2$

Below $Pe = (Re.Pr) = 0.2$, following relation is recommended by Nakai and Okazaki:

$$Nu_{cyl} = \left(0.8237 - \ln\left(Pe^{\frac{1}{2}}\right)\right)^{-1} \qquad (\text{for } Pe < 0.2 \text{...(9.92)})$$

For Eqs. 9.91 and 9.92 also, properties are evaluated at the film temperature.

For heat transfer from a single cylinder in cross flow, **for liquid metals**, following relation is recommended by Ishiguro et. al.:

$$Nu_{cyl} = 1.125 \cdot (Re \cdot Pr)^{0.413} \qquad (\text{for } 1 < Re.Pr < 100 \text{...(9.93)})$$

However, note that Eq. 9.90 is quite comprehensive and is also valid for liquid metals.

For circular cylinder in cross flow, **for gases**, following relation is widely used:

$$Nu = C \cdot Re^n \cdot Pr^{\frac{1}{3}} \qquad \text{...(9.94)}$$

where, values of C and n are given in Table 9.5:

TABLE 9.5 Values of C and *n* in Eq. 9.94

Re	C	n
0.4–4	0.989	0.330
4–40	0.911	0.385
40–4,000	0.683	0.466
4,000–40,000	0.193	0.618
40,000–400,000	0.0266	0.805

All fluid properties are taken at film temperature.

For non-circular cylinders:

Again, Eq. 9.94 is applicable.

For non-circular cylinders, Fig. 9.17 below, gives the values of C and n to be used in Eq. 9.94. This figure also shows the characteristic dimension 'D' used to calculate the Reynolds number, for each geometry.

Flow across spheres:

For gases, McAdams recommends following relation:

$$Nu_{sph} = 0.37 \cdot Re^{0.6} \quad \text{(for } 25 < Re < 100{,}000\text{...(9.95))}$$

For flow of liquids past spheres, Kramers suggests following relation:

$$Nu_{sph} \cdot Pr^{-0.3} = 0.97 + 0.68 \cdot Re^{0.5} \quad \text{(for } 1 < Re < 2000\text{...(9.96))}$$

In Eq. 9.95 and 9.96, fluid properties are evaluated at film temperature.

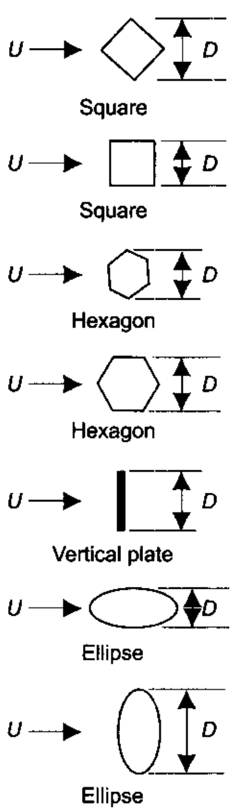
A comprehensive equation **for gases and liquids** flowing past a sphere is given by Whitaker:

$$Nu_{sph} = 2 + \left(0.4 \cdot Re^{\frac{1}{2}} + 0.06 \cdot Re^{\frac{2}{3}}\right) \cdot Pr^{0.4} \cdot \left(\frac{\mu_a}{\mu_w}\right) \quad \text{...(9.97)}$$

Eq. 9.97 is valid for: $3.5 < Re < 80{,}000$ and $0.7 < Pr < 380$. Here, fluid properties are evaluated at free stream temperature.

A special case is that of heat and mass transfer from **freely falling liquid drops** and the following correlation of Ranz and Marshall is applicable:

$$Nu_{avg} = 2 + 0.6 \cdot Re^{\frac{1}{2}} \cdot Pr^{\frac{1}{3}} \quad \text{...(9.97a)}$$



Shape	Re	C	n
Square	$5 \times 10^3 - 10^5$	0.246	0.588
Square	$5 \times 10^3 - 10^5$	0.102	0.675
Hexagon	$5 \times 10^3 - 1.95 \times 10^4$	0.16	0.638
	$1.95 \times 10^4 - 10^5$	0.0385	0.782
Hexagon	$5 \times 10^3 - 10^5$	0.153	0.638
Vertical plate	$4 \times 10^3 - 1.5 \times 10^4$	0.228	0.731
Ellipse	$2.5 \times 10^3 - 1.5 \times 10^4$	0.224	0.612
Ellipse	$3 \times 10^3 - 1.5 \times 10^4$	0.085	0.804

FIGURE 9.17 Constants C and n for cross flow over noncircular cylinders

For heat transfer from a sphere to a **liquid metal**, following correlation is recommended:

$$Nu_{sph} = 2 + 0.386\cdot(Re\cdot Pr)^{\frac{1}{2}} \qquad (\textit{for } 36{,}000 < Re < 200{,}000...(9.98))$$

In Eq. 9.98, fluid properties are to be evaluated at film temperature.

9.9.2 *Flow Across Bluff Objects*

Normal flat plate (width *D*):

$$Nu_D = 0.20\cdot Re_D^{\frac{2}{3}} \qquad (\textit{for } 1 < Re < 4\times 10^5...(9.99))$$

Half round cylinder of dia. *D*, with flat surface at rear:

$$Nu_D = 0.16\cdot Re_D^{\frac{2}{3}} \qquad (\textit{for } 1 < Re < 4\times 10^5...(9.100))$$

In Eqs. 9.99 and 9.100, fluid properties are evaluated at film temperature.

9.9.3 *Flow Through Packed Beds*

Here, a gas or liquid flows through a bed packed with solid particles (such as spheres, cylinders or commercial packings like Raschig rings, ceramic saddles etc.). During the 'charging cycle', the hot fluid, while passing through the bed, gives up its 'heat' to the solid particles, and during the 'discharge cycle', the incoming cooler fluid picks up the stored heat from the solid particles.

Packed beds are used in catalytic reactors, grain dryers, storage of solar thermal energy, gas chromatography, regenerators and desiccant beds.

Reynolds number in the correlations is based on a 'superficial velocity' U_s, i.e. the fluid velocity that would exist if the bed were empty. Characteristic length used is the equivalent diameter of the packing, D_p. Another parameter that appears in some correlations is the void fraction, ε, i.e. the fraction of bed volume that is empty.

Whitaker recommends following relation for **heat transfer between the gas and packings** (including cylinders with diameter equal to height, spheres, or several types of commercial packings such as Raschig rings, partition rings or Berl saddles):

$$\frac{h_a\cdot D_p}{k} = \frac{1-\varepsilon}{\varepsilon}\cdot\left(0.5\cdot Re_{D_p}^{\frac{1}{2}} + 0.02\cdot Re_{D_p}^{\frac{2}{3}}\right)\cdot Pr^{\frac{1}{3}} \qquad ...(9.101)$$

where h_a is the average heat transfer coefficient

Eq. 9.101 is valid for: $20 < Re_{Dp} < 10{,}000$, and $0.34 < \varepsilon < 0.78$.

Packing diameter D_p is defined as six times the volume of the particle divided by the particle surface area; for a sphere, D_p = diameter of sphere. All properties are evaluated at bulk fluid temperature (One may use the average of inlet and outlet temperature of the heat exchanger). In the above correlation, Reynolds number is defined as:

$$Re_{Dp} = \frac{D_p\cdot U_s}{\nu\cdot(1-\varepsilon)}$$

Eq. 9.101 does not correlate data well for cube packings.

To determine the heat transfer from the **wall of a packed bed to a gas**, Beek recommends the following relation, for particles like cylinders, which can pack next to the wall:

$$\frac{h_a\cdot D_p}{k} = 2.58\cdot Re_{Dp}^{\frac{1}{3}}\cdot Pr^{\frac{1}{3}} + 0.094\cdot Re_{Dp}^{0.8}\cdot Pr^{0.4} \qquad ...(9.102)$$

and, for particles like spheres, which contact the wall at one point:

$$\frac{h_a\cdot D_p}{k} = 0.203\cdot Re_{Dp}^{\frac{1}{3}}\cdot Pr^{\frac{1}{3}} + 0.220\cdot Re_{Dp}^{0.8}\cdot Pr^{0.4} \qquad ...(9.103)$$

In Eqs. 9.102 and 9.103, properties of fluid are evaluated at the film temperature. Also, the Reynolds number is:

$$40 < Re_{Dp} = \frac{U_s\cdot D_p}{\nu} < 2000$$

where D_p is the diameter of sphere or cylinder. For other types of packings, Whitaker's definition of D_p may be used.

Beek also gives the correlation for the friction factor:

$$f = \frac{D_p}{L} \cdot \frac{\Delta p}{\rho \cdot U_s^2} = \frac{1-\varepsilon}{\varepsilon^3} \cdot \left(1.75 + 150 \cdot \frac{1-\varepsilon}{Re_{Dp}}\right) \quad \text{...(9.104)}$$

where Δp is the pressure drop over a length L of the packed bed.

Example 9.12. Air at 35°C flows across a cylinder of 50 mm diameter at a velocity of 50 m/s. The cylinder surface is maintained at 145°C. Find the heat loss per unit length. Properties at mean temperature of 90°C are: $\rho = 1$ kg/m³, $\mu = 20 \times 10^{-6}$ kg/(ms), $k = 0.0312$ W/(mC), $C_p = 1.0$ kJ/(kgC).

Use the relation: $Nu_D = 0.027.(Re_D)^{0.805}.(Pr)^{1/3}$ [M.U.]

Solution.

Data:

$T_s := 145°C \quad T_a := 35°C \quad V := 50$ m/s $\quad D := 0.05$ m $\quad L := 1$ m (assumed) $\quad T_f := \frac{T_a + T_s}{2}$ i.e. $T_f = 90°C$

Properties at T_f:

$\mu := 20 \times 10^{-6}$ kg/(ms) $\quad k := 0.0312$ W/(mC) $\quad C_p := 1000$ J/(kgC) $\quad \rho := 1$ kg/m³ $\quad A := \pi \cdot D \cdot L$ m²

i.e. $A = 0.157$ m²

Reynolds number:

$$Re := \frac{D \cdot V \cdot \rho}{\mu}$$

i.e. $Re = 1.25 \times 10^5$

Prandtl number:

$$Pr := \frac{C_p \cdot \mu}{k}$$

i.e. $Pr = 0.641$

Nusselts number:

We have:

$$Nu_D := 0.027 \cdot Re^{0.805} \cdot Pr^{\frac{1}{3}}$$

i.e. $Nu_D = 295.122$

Therefore, heat transfer coefficient:

$$h := \frac{k \cdot Nu_D}{D}$$

i.e. $h := 184.156$ ...W/(m²C).

Heat transferred, Q:

$$Q := h \cdot A \cdot (T_s - T_a)$$

i.e. $Q = 3.182 \times 10^3$ **W**

Example 9.13. A hot wire probe is 5 mm in length, 10 μm diameter wire with an electrical resistance of 150 ohms/m. The wire is maintained at a constant temperature of 50°C. If the the probe is kept in an air stream flowing at a velocity of 10 m/s and at 1 bar and 25°C, determine the current required to maintain the wire temperature at 50°C.

Solution.

Data:

$T_s := 50°C \quad T_a := 25°C \quad U := 10$ m/s $\quad D := 10 \times 10^{-6}$ m $\quad L := 0.005$ m $\quad T_f := \frac{T_a + T_s}{2}$ i.e. $T_f = 37.5°C$

Properties at T_f:

$\nu := 16.7 \times 10^{-6}$ m²/s $\quad k := 0.02704$ W/(mK) $\quad Pr := 0.706$

Reynolds number:

$$Re := \frac{U \cdot D}{\nu} \quad \text{i.e.} \quad Re = 5.988$$

Therefore, $Re \cdot Pr = 4.228$

Since $Re.Pr > 0.2$, we can use the correlation of Churchill and Bernstein, viz.

$$Nu_{cyl} := \left[0.3 + \frac{0.62 \cdot Re^{\frac{1}{2}} \cdot Pr^{\frac{1}{3}}}{\left[1 + \left(\frac{0.4}{Pr} \right)^{\frac{2}{3}} \right]^{\frac{1}{4}}} \cdot \left[1 + \left(\frac{Re}{28200} \right)^{\frac{5}{8}} \right]^{\frac{4}{5}} \right] \quad ...(9.90)$$

i.e. $Nu_{cyl} = 1.491$ *(Nusselt number)*

Therefore, heat transfer coefficient:

$$h := \frac{Nu_{cyl} \cdot k}{D}$$

i.e. $h = 4.031 \times 10^3$ W/(m^{2}C) *(heat transfer coefficient)*

Heat transferred Q:

$$Q := h \cdot (\pi \cdot D \cdot L) \cdot (T_s - T_a) \text{ W}$$

i.e. $Q = 0.016$ W *(heat dissipated = 16 mW)*

This is also equal to the value of electrical power dissipated; $Q = I^2.R$

$R := 150.0.005$ ohms *(electrical resistance of the wire)*

i.e. $R = 0.75$ ohms

Therefore, current flow required:

$$I := \sqrt{\frac{Q}{R}} \text{ Amp}$$

i.e. $I = 0.145$ **A** *(**current flow required.**)*

Alternatively:

To calculate *Nu* we can also use Eq. 9.94:

$$Nu = C \cdot Re^n \cdot Pr^{\frac{1}{3}} \quad ...(9.94)$$

Then, for circular cylinder, we get for $Re = 5.988$, from the Table 9.5:

$C := 0.911$ and $n = 0.385$

Therefore, $Nu := C \cdot Re^n \cdot Pr^{\frac{1}{3}}$

i.e. $Nu = 1.616$

And,

$$h := \frac{Nu \cdot k}{D}$$

i.e. $h = 4.369 \times 10^3$ W/(m^{2}C) *(heat transfer coefficient)*

Therefore, $Q: = h \cdot (\pi \cdot D \cdot L) \cdot (T_s - T_a)$ W

i.e. $Q = 0.017$ W *(heat dissipated = 17 mW)*

This value is almost the same as obtained by the correlation of Churchill and Bernstein.

Therefore, current flow required:

$$I := \sqrt{\frac{Q}{R}} \text{ Amp}$$

i.e. $I = 0.151$ **A** *(**current flow required.**)*

Example 9.14. Air at 25°C flows across an elliptical tube 6 cm × 12 cm size, perpendicular to the minor axis with a velocity of 3 m/s. Tube surface is maintained at 55°C. Determine the value of convection coefficient.

Solution.

Data:

$T_s := 55°C$ $\quad T_a := 25°C$ $\quad U := 3$ m/s $\quad D_1: = 0.06$ m $\quad D_2 := 0.12$ m $\quad T_f := \frac{T_a + T_s}{2}$ i.e. $T_f = 40°C$

Properties at $T_f = 40°C$:

$\nu := 17.6 \times 10^{-6}\ m^2/s$ $k := 0.0265\ W/(mK)$ $Pr := 0.71$

Reynolds number:

See Fig. 9.17 for the case of flow across an ellipse.

$$Re := \frac{U \cdot D_1}{\nu} \quad \text{i.e.} \quad Re = 1.023 \times 10^4$$

Then, we use Eq. 9.94, viz.

$$Nu = C \cdot Re^n \cdot Pr^{\frac{1}{3}} \qquad ...(9.94)$$

Values of C and n are obtained from Fig. 9.17 as:

$$C := 0.224 \quad \text{and,} \quad n: = 0.612$$

Therefore, $$Nu := C \cdot Re^n \cdot Pr^{\frac{1}{3}}$$

i.e. $$Nu = 56.838$$

Heat transfer coefficient:

Therefore, $$h = \frac{Nu \cdot k}{D_1}$$

i.e. $$h = 25.104\ \mathbf{W/(m^2K)}$$ ***...heat transfer coefficient.***

Example 9.15. In a packed bed heat exchanger, air is heated from 40°C to 360°C by passing it through a 10 cm diameter pipe, packed with spheres of 8 mm diameter. The flow rate is 18 kg/h. Pipe surface temperature is maintained at 400°C. Determine the length of bed required.

Solution.

Data:

$T_s := 400°C$ $T_{in} := 40°C$ $T_{out}: = 360°C$ $D: = 0.008\ m$ $d_{pipe}: = 0.1\ m$

$m_{air} := \frac{18}{3600}$ i.e. $m_{air} = 5 \times 10^{-3}\ kg/s$

Average air temperature = (40 + 360)/2 = 200°C

Therefore, average film temperature = (200 + 400)/2 = 300°C

Taking properties of air at 300°C:

$\rho := 0.596\ kg/m^3$ $C_p := 1047\ J/(kgK)$ $k := 0.0429\ W/(mK)$ $\nu := 49.2 \times 10^{-6}\ m^2/s$ $Pr := 0.71$

Equivalent particle diameter = 6 × volume/surface area = D for a sphere

i.e. $$D_p := 0.008\ m$$ *(equivalent particle diameter.)*

Therefore, superficial velocity:

$$U_s := \frac{m_{air}}{\rho \cdot \frac{\pi \cdot d_{pipe}}{4}}$$

i.e. $$U_s = 1.068\ m/s$$ *(superficial velocity)*

Reynolds number:

Therefore,

$$Re_{Dp} := \frac{U_s \cdot D_p}{\nu}$$

i.e. $$Re_{Dp} = 173.684$$

Nusselts number:

We use Eq. 9.103, viz.

$$\frac{h_a \cdot D_p}{k} := 0.203 \cdot Re_{Dp}^{\frac{1}{3}} \cdot Pr^{\frac{1}{3}} + 0.220 \cdot Re_{Dp}^{0.8} \cdot Pr^{0.4} \qquad ...(9.103)$$

$$\text{for } 40 < Re_{Dp} = \frac{U_s \cdot D_p}{\nu} < 2000$$

i.e. $$\frac{h_a \cdot D_p}{k} = 12.888$$

Heat transfer coefficient

$$h_a := \frac{12.888 \cdot k}{D_p} \ \text{W/(m}^2\text{K)} \qquad \textit{(heat transfer coefficient)}$$

i.e. $$h_a = 69.112 \ \text{W/(m}^2\text{K)} \qquad \textit{(heat transfer coefficient)}$$

Now, heat gained by air, Q = heat transfer between the wall surface and air

i.e. $$Q := m_{air} \cdot C_p \cdot (T_{out} - T_{in}) \ \text{W}$$

i.e. $$Q = 1.675 \times 10^3 \ \text{W} \qquad \textit{(heat gained by air)}$$

This should be equal to heat transfer between the wall surface and air = $h_a x$ pipe surface area x LMTD

Here, LMTD is the 'log mean temperature difference' between the pipe surface and the air stream. Since the temperature of air stream goes on changing along the length of heat exchanger, we use a mean temperature difference between the pipe surface and this air stream, given by LMTD.

LMTD is defined as follows:

$$\text{LMTD} = \frac{\Delta T_{max} - \Delta T_{min}}{\ln\left(\frac{\Delta T_{max}}{\Delta T_{min}}\right)} \qquad \textit{(see chapter on Heat exchangers for derivation of this equation for LMTD)}$$

$$\Delta T_{max} := T_s - T_{in} \quad \text{i.e.} \quad \Delta T_{max} = 360°\text{C}$$

$$\Delta T_{min} := T_s - T_{out} \quad \text{i.e.} \quad \Delta T_{min} = 40°\text{C}$$

Therefore, $$\text{LMTD} := \frac{\Delta T_{max} - \Delta T_{min}}{\ln\left(\frac{\Delta T_{max}}{\Delta T_{min}}\right)}$$

i.e. $$\text{LMTD} = 145.638°\text{C}$$

Now, writing the heat balance, $Q = h_a X$ pipe surface area x LMTD , we get:

$$Q = h_a \cdot (\pi \cdot d_{pipe} \cdot L) \cdot \text{LMTD}$$

where L is the length of pipe (= height of bed)

i.e. $$L := \frac{Q}{h_a \cdot \pi \cdot d_{pipe} \cdot \text{LMTD}} \ \text{m} \qquad \textit{(length of bed)}$$

i.e. $$L = 0.53 \ \mathbf{m} \qquad \textit{(length of bed.)}$$

9.9.4 Flow Across a Bank of Tubes

Flow across a bank of tubes is practically a very important case. In many industrial heat exchangers, one of the fluids flows inside the tubes in a shell and the second fluid flows through the shell, across the tubes. Typical applications are: in water tube boilers where water flows through the tubes and hot flue gases flow across these tubes, waste heat recovery systems, air conditioning applications and common 'shell and tube' heat exchangers used in numerous industrial applications.

Tubes in a tube bank may be arranged either in an 'in-line' configuration or in a 'staggered' configuration, as shown in Fig. 9.18. In the figure, S_L is the 'longitudinal pitch', S_T is the 'transverse pitch' and S_D is the 'diagonal pitch'.

Zhukauskas (1972) proposed the following correlation for Nusselts number, based on a large amount of experimental data:

$$Nu_a = \frac{h_a \cdot D}{k} = C \cdot (Re_D)^m \cdot Pr^{0.36} \cdot \left(\frac{\text{Pr}}{\text{Pr}_w}\right)^{0.25} \qquad ...(9.105)$$

where Nu_a is the average Nusselts number

h_a is the average heat transfer coefficient

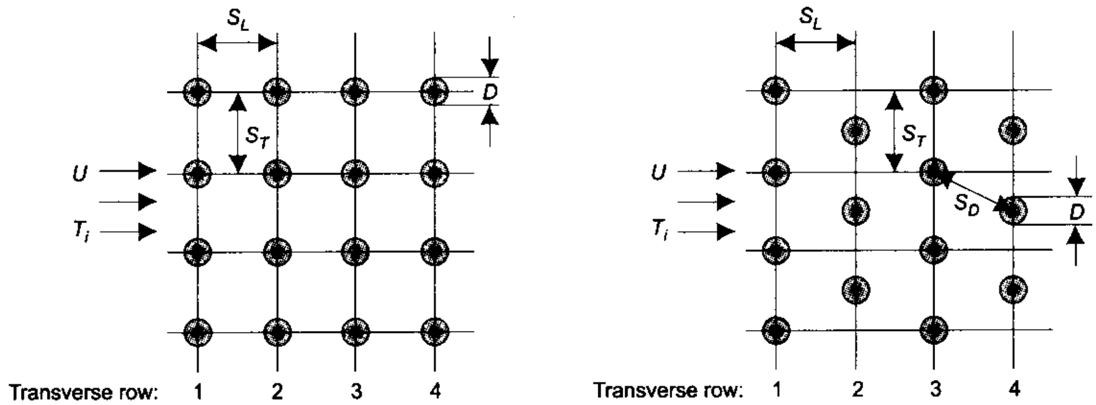
$Re_D = (d \ \rho \ U_{max})/\mu$

Pr is the bulk Prandtl number

Pr_w is the wall Prandtl number

And,

$$U_{max} = \frac{S_T}{S_T - D} \cdot U \qquad \textit{(for aligned arrangement...(9.106))}$$



(a) Aligned arrangement of tubes (a) Staggered arrangement of tubes

FIGURE 9.18 Flow across a tube bank

and,
$$U_{max} = \frac{S_T}{2 \cdot (S_D - D)} \cdot U \quad \text{(for staggered arrangement...(9.107))}$$

Note: (i) While calculating U_{max} for the staggered arrangement, calculate with both the Eqs. 9.106 and 9.107 and adopt the larger value so obtained. U is the velocity of fluid as it approaches the tube bank.

(ii) For gases, Prandtl number ratio may be dropped, since it does not have much influence

(iii) All properties (except Pr_w) are evaluated at free stream temperature

Eq. 9.105 gives very good prediction when the number of tube rows in the bank,

$N > 20$, and $0.7 < Pr < 500$, and $1000 < Re_{D\_max} < 2 \times 10^6$. However, the equation can be used even when $N < 20$, with a correction factor applied. If $N = 4$, error involved in prediction is about 25%.

Eq. 9.105 takes the following forms for various flow regimes:

For Laminar flow (i.e. $10 < Re_D < 100$):

$$Nu_a = 0.8 \cdot Re_D^{0.4} \cdot Pr^{0.36} \cdot \left(\frac{Pr}{Pr_w}\right)^{0.25} \quad \text{(for In-line tubes...(9.108))}$$

and,
$$Nu_a = 0.9 \cdot Re_D^{0.4} \cdot Pr^{0.36} \cdot \left(\frac{Pr}{Pr_w}\right)^{0.25} \quad \text{(for staggered tubes...(9.109))}$$

These equations have been validated also in the range: $50 < Re_D < 1000$.

For transition regime (i.e. $1000 < Re_D < 2 \times 10^5$):

$$Nu_a = 0.27 \cdot Re_D^{0.63} \cdot Pr^{0.36} \cdot \left(\frac{Pr}{Pr_w}\right)^{0.25} \quad \text{(for In-line tubes, } S_T/S_L > 0.7\text{...(9.110))}$$

Note: $S_T/S_L < 0.7$ for in-line tubes, gives very ineffective heat exchanger and should not be used.

$$Nu_a = 0.35 \cdot \left(\frac{S_T}{S_L}\right)^{0.2} \cdot Re_D^{0.60} \cdot Pr^{0.36} \cdot \left(\frac{Pr}{Pr_w}\right)^{0.25} \quad \text{(for staggered tubes, } S_T/S_L < 2\text{...(9.111))}$$

and,
$$Nu_a = 0.40 \cdot Re_D^{0.60} \cdot Pr^{0.36} \cdot \left(\frac{Pr}{Pr_w}\right)^{0.25} \quad \text{(for staggered tubes, } S_T/S_L \text{ greater than or equal to 2...(9.112))}$$

For turbulent regime (i.e. $Re_D > 2 \times 10^5$):

$$Nu_a = 0.021 \cdot Re_D^{0.84} \cdot Pr^{0.36} \cdot \left(\frac{Pr}{Pr_w}\right)^{0.25}$$ ...*for In-line tubes*...(9.113))

$$Nu_a = 0.022 \cdot Re_D^{0.84} \cdot Pr^{0.36} \cdot \left(\frac{Pr}{Pr_w}\right)^{0.25}$$ (*for staggered tubes, Pr* > 1...(9.114))

and, $$Nu_a = 0.019 \cdot Re_D^{0.84}$$ (*for staggered tubes, Pr* = 0.7...(9.115))

For staggered arrangement, with $S_T/D = 2$ and $S_L/D = 1.4$, we have the relation due to Achenbach:

$$Nu_a = 0.0131 \cdot Re_D^{0.883} \cdot Pr^{0.36}$$ ...(9.116)

Eq. 9.116 is valid in the range $4.5 \times 10^5 < Re_D < 7 \times 10^6$.

If the number of tube rows is < 20, a correction factor is applied to the calculated Nusselts number as follows:

$$Nu_{a\_N} = Nu_a \cdot C_2$$ ...(9.117)

where Nu_{a_N} is the Nusselts number for the actual tube bank with $N < 20$, and

Nu_a is the value of Nusselts number calculated for $N > 20$, using one of the appropriate relations given above

C_2 is the correction factor taken from Table 9.6.

TABLE 9.6 Correction factor C_2 in Eq. 9.117 for $N < 20$

N	*1*	*2*	*3*	*4*	*5*	*7*	*10*	*13*	*16*
Alinged	0.70	0.80	0.86	0.90	0.92	0.95	0.97	0.98	0.99
Staggered	0.64	0.76	0.84	0.89	0.92	0.95	0.97	0.98	0.99

Pressure drop: Pressure drop (in Pascals) for flow of gases over a bank of tubes is given by:

$$\Delta p = \frac{2 \cdot f' \cdot G_{max}^2 \cdot N}{\rho} \cdot \left(\frac{\mu_w}{\mu_b}\right)^{0.14}$$ *Pa*...(9.118).

where G_{max} = mass velocity at minimum flow area = $\rho.U_{max}$

ρ = density, evaluated at free stream conditions

N = number of transverse rows

μ_b = average free stream viscosity

Friction factor, f' is given by:

$$f' = \left[0.25 + \frac{0.118}{\left[\frac{(S_T - D)}{D}\right]^{1.08}}\right] \cdot Re_D^{-0.16}$$ (*for staggered tubes*...(9.119))

and,

$$f' = \left[0.044 + \frac{0.08 \cdot \left(\frac{S_L}{D}\right)}{\left[\frac{(S_T - D)}{D}\right]^{0.43 + 1.13 \cdot \frac{D}{S_L}}}\right] \cdot Re_D^{-0.15}$$ (*for in-line tubes*...(9.120))

Example 9.16. Air at 1 bar and 20°C flows across a bank of tubes 10 rows high and 4 rows deep; air velocity is 8 m/s, measured at the entry to the tube bank. Diameter of the tubes is 25 mm and surface temperature of the tubes is maintained at 80°C. Tubes are arranged in an in - line manner. $S_T = S_L = 37.5$ mm. Calculate the total heat transfer per unit length of the tube bank, and also the exit air temperature. Also, find out the pressure drop.

Solution.

Data:

$T_s := 80°C \quad T_i := 20°C \quad U := 8\ m/s \quad S_T := 0.0375\ m \quad S_L := 0.0375\ m \quad D := 0.025\ m$

Reyonlds number:

This is based on U_{max}. We have:

$$U_{max} := \frac{S_T}{S_T - D} \cdot U$$ *(for aligned arrangement...(9.106))*

i.e. $U_{max} = 24\ m/s$ *(maximum velocity)*

Taking properties of air at free stream temperature of 20°C:

$\rho := 1.164\ kg/m^3 \quad C_p := 1012\ J/(kgK) \quad k := 0.0251\ W/(mK) \quad \nu := 15.7 \times 10^{-6}\ m^2/s \quad Pr := 0.71$

Therefore, $$Re_D := \frac{U_{max} \cdot D}{\nu}$$

i.e. $Re_D = 3.822 \times 10^4$ *(Reynolds number)*

Nusselts number:

Since Reynolds number is between 1000 and 200,000 which is in the transition regime, the appropriate equation for average Nusselts number is:

$$Nu_a = 0.27 \cdot Re_D^{0.63} \cdot Pr^{0.36} \cdot \left(\frac{Pr}{Pr_w}\right)^{0.25}$$ *...for in-line tubes, $S_T/S_L > 0.7$...(9.110)*

The last term, i.e. the ratio of Prandtl numbers can be neglected for gases: So, we have:

$$Nu_a := 0.27 \cdot Re_D^{0.63} \cdot Pr^{0.36}$$

i.e. $Nu_a = 183.923$ *(Nusselts number)*

Therefore, average heat transfer coefficient is:

$$h_a := \frac{Nu_a \cdot k}{D}\ W/(m^2C)$$ *(average heat transfer coefficient)*

i.e. $h_a = 184.658\ W/(m^2C)$ *(average heat transfer coefficient ($N > 20$))*

This is the value of heat transfer coefficient that would be obtained if there were 20 rows of tubes in the direction of flow. But, in the present case, there are only 4 rows in the direction of flow. So, from the Table, we get the correction factor as:

$$C_2 = 0.90$$

Therefore, actual heat transfer coefficient is = 184.658 × 0.9 *(actual average heat transfer coefficient)*

i.e. $h_a = 166.193\ W/(m^2C)$ *(actual average heat transfer coefficient)*

Surface area for heat transfer for unit length of tubes is:

$$A := (10 \cdot 4) \cdot (\pi \cdot D \cdot 1)\ m^2/m$$ *(for 10 rows high, 4 rows deep)*

i.e. $A = 3.142\ m^2/m.$

Total heat transfer rate Q:

Now, total heat transfer rate is given by Newton's law:

$$Q = h_a \cdot A \cdot \Delta \cdot T$$

Here ΔT is the average temperature difference between the wall and the air stream. However, temperature of air stream goes on changing from entry to exit in the heat exchanger. So, we use a 'mean temperature difference' called LMTD (log mean temperature difference). Expression for LMTD is derived in the chapter on heat exchangers. For the present, let us take for LMTD:

$$LMTD = \frac{(T_s - T_i) - (T_s - T_o)}{\ln\left(\frac{T_s - T_i}{T_s - T_o}\right)}$$

We need the exit temperature T_o of the air stream. This is calculated by a heat balance:

$$Q = h_a \cdot A.(LMTD)$$

mass_flow: $= \rho \cdot U \cdot 10 \cdot S_T$ ...kg/s *(mass flow rate; 10 rows high S_T is transverse distance)* i.e. $\text{mass\_flow} = 3.492$ kg/s

Then, we can write the heat balance:

$$h \cdot A \cdot \text{LMTD} = \text{mass\_flow} \cdot C_p \cdot (T_o - T_i)$$

Substitute for LMTD and solve for T_o.

Use Solve block of Mathcad; assume a guess value for T_o to start with, say T_o = 70°C. Then type 'Given' and write the constraint; then type Find (T_o) and get the answer:

$$T_o := 70 \qquad \textit{(guess value)}$$

Given

$$h_a \cdot A \cdot \frac{(T_s - T_i) - (T_s - T_o)}{\ln\left(\frac{T_s - T_i}{T_s - T_o}\right)} = \text{mass\_flow} \cdot C_p \cdot (T_o - T_i)$$

$$\text{Find}\ (T_o) = 28.241$$

i.e. $T_o = 28.241$°C *(exit air temperature)*

Therefore, heat transfer rate, Q:

$$Q := h_a \cdot A \cdot \frac{(T_s - T_i) - (T_s - T_o)}{\ln\left(\frac{T_s - T_i}{T_s - T_o}\right)}$$

i.e. $Q = 2.912 \times 10^4$ **W/m = 29.12 kW/m**

Alternatively, we can use the arithmetic average value of air stream between the inlet and outlet temperature, since this is simpler to calculate and error involved will not be much:

Then, $$Q = h_a \cdot A \cdot \left[T_s - \left(\frac{To + T_i}{2}\right)\right] = \text{mass\_flow} \cdot C_p \cdot (T_o - T_i)$$

Using Solve block as earlier, to obtain T_o:

$$T_o := 70 \qquad \textit{(guess value)}$$

Given

$$Q = h_a \cdot A \cdot \left[T_s - \left(\frac{To + T_i}{2}\right)\right] = \text{mass\_flow} \cdot C_p \cdot (T_o - T_i)$$

$$\text{Find}\ (T_o) = 28.255$$

i.e. $T_o = 28.255$°C *(exit air temperature)*

i.e. we get practically the same value for To as obtained earlier.

And, $$Q := h_a \cdot A \cdot \left[T_s - \left(\frac{T_o + T_i}{2}\right)\right] \text{ W/m}$$

i.e. $Q = 2.917 \times 10^4$ **W/m = 29.17 kW/m.**

Pressure drop:

We have:

$$\Delta p = \frac{2 \cdot f' \cdot G_{max}^2 \cdot N}{\rho} \cdot \left(\frac{\mu_w}{\mu_b}\right)^{0.14} \text{ Pa} \qquad \text{...(9.118)}$$

$$G_{max} := \rho \cdot U_{max} \text{ kg/s.m}^2 \qquad \textit{(mass velocity)}$$

i.e. $G_{max} = 27.936$ kg/s.m² *(mass velocity)*

$N = 4$ *(number of transverse rows)*

$$f' := \left[0.044 + \frac{0.08 \cdot \left(\frac{S_L}{D}\right)}{\left[\frac{S_T - D}{D}\right]^{0.43 + 1.13 \cdot \frac{D}{S_L}}}\right] \cdot Re_D^{-0.15} \qquad \textit{(for in-line tubes...(9.120))}$$

i.e. $f' = 0.065$ *(friction factor)*

$\mu_w := 20.79 \times 10^{-6}$ kg/ms (*dynamic viscosity of air at 80°C*)

$\mu_b := 18.46 \times 10^{-6}$ kg/ms (*dynamic viscosity of air at average free stream temperature of 24.5°C*)

Therefore,

$$\Delta p := \frac{2 \cdot f' \cdot G_{max}^2 \cdot N}{\rho} \cdot \left(\frac{\mu_w}{\mu_b}\right)^{0.14} \text{ Pa}$$

i.e. $\Delta p = 354.613$ **Pa = 0.003546 bar**

9.10 Flow Inside Tubes

Circular tubes are the most commonly used geometry for cooling and heating applications, in industry. Often, tubes of other geometries such as square or rectangle are also used. We are interested in heat transfer in such cases; pressure drop occurring during flow is also of interest since it has a direct bearing on the pumping power required to cause the flow.

Observe the major difference between the external flows just studied and the internal flow through pipes: in the external flow, say over a flat plate, there was a free surface of fluid and the boundary layer was free to grow indefinitely; however, in a pipe flow, the flow is confined within the pipe and the boundary layer growth is limited to grow only upto the centre of the pipe.

9.10.1 Hydrodynamic and Thermal Boundary Layers for Flow in a Tube

Consider a fluid entering into a circular pipe, with a uniform velocity U (See Fig. 9.19). Fluid layer coming in contact with the pipe surface comes to a complete halt and the adjacent layers slow down gradually due to viscosity effects. Since the total mass flow in a section must remain constant, velocity in the central portion increases. As a result, a 'velocity boundary layer' develops along the pipe. Thickness of the velocity boundary layer increases along the flow length until the entire pipe is filled up with the boundary layer, as shown. 'Hydrodynamic entry length (L_h)' is the distance from the entry point to the point where the boundary layer has developed upto the centre. In the region beyond the hydrodynamic entry length, the velocity profile is fully developed and remains unchanged; this is the 'hydrodynamically developed region'. As will be shown later, velocity profile in the fully developed region, in laminar flow, is parabolic; in turbulent flow, the velocity profile is a truncated one.

Similarly, when a fluid at an uniform temperature enters a pipe whose wall is at different temperature, a 'thermal boundary layer ' develops along the pipe. Thickness of thermal boundary layer also increases along the flow length till the boundary layer reaches the centre of the pipe. 'Thermal entry length (L_t)' is the distance from the entry to the point where the thermal boundary layer has reached the centre, and is shown in the Fig. 9.19. Beyond this point, along the length, we have the 'fully developed flow' i.e. the flow is both hydrodynamically and thermally fully developed.

Temperature profile may vary with x even in the thermally developed region. However, the dimensionless temperature profile expressed as $(T - T_s)/(T_m - T_s)$ remains constant in the thermally developed region, whether the temperature of the pipe surface remains constant or the heat flux at the surface remains constant. (Tm is the bulk or mean temperature at a given section).

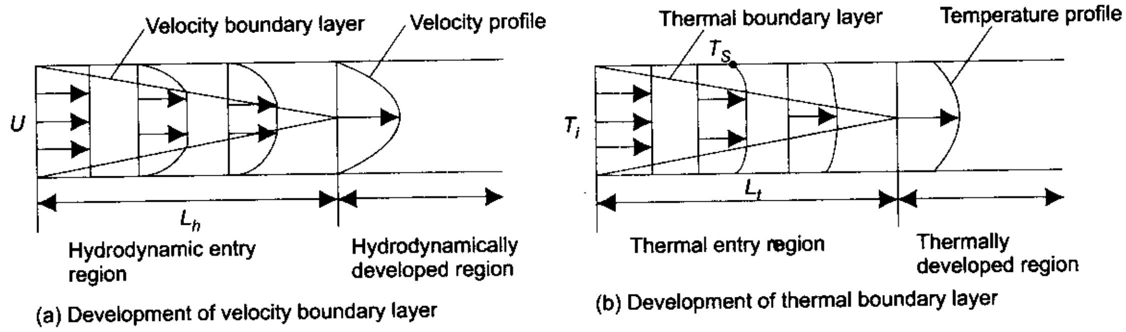


FIGURE 9.19 Flow inside a pipe

Relative growth of hydrodynamic and thermal boundary layers is controlled by the dimensionless Prandtl number. For gases, $Pr = 1$, and the hydrodynamic and thermal boundary layers essentially coincide; for oils $Pr >> 1$ and the hydrodynamic boundary layer outgrows the thermal boundary layer, i.e. hydrodynamic entry length is smaller for oils. For fluids with $Pr << 1$, such as liquid metals, thermal boundary layer outgrows the hydrodynamic boundary layer and consequently, the thermal entry length is shorter than the hydrodynamic entry length.

Reynolds number is the dimensionless number that characterizes the flow inside a tube as laminar or turbulent. Reynolds number is defined as:

$$Re = (U_m.D)/\nu$$

where U_m is the mean velocity in the pipe, and ν is the kinematic viscosity of the fluid. Flow regimes are defined as follows, depending upon the Reynolds number:

$$Re < 2300 \qquad \textit{(Laminar flow)}$$

$$2300 \le Re \le 4000 \qquad \textit{(Transition flow)}$$

$$Re > 4000 \qquad \textit{(Turbulent flow)}$$

Hydrodynamic and thermal entry lengths:

In laminar flow:

$$L_{h\_lam} = 0.05 \cdot Re \cdot D \qquad ...(9.121a)$$

$$L_{t\_lam} = 0.05 \cdot Re \cdot Pr \cdot D \qquad ...(9.121b)$$

In turbulent flow, hydrodynamic and thermal entry lengths are independent of *Re* and *Pr* and are generally taken to be:

$$L_{h\_turb} = L_{t\_turb} = 10 \cdot D \qquad ...(9.122)$$

The friction coefficient or shear stress at the surface is related to the slope of the velocity profile at the surface. Since the velocity profile remains essentially constant in the hydrodynamically developed region, the friction factor and the shear stress remain constant in the hydrodynamically developed region. By a similar argument, heat transfer coefficient also remains constant in the thermally developed region.

At the entry to the tube, thickness of the boundary layer is practically zero; so velocity and temperature gradients at the surface are almost infinite at the entry, which means that the heat transfer coefficient and pressure drop are the highest in the entry region and go on decreasing along the length.

Generally, in practice, turbulent flows prevail in heat transfer applications; length of pipes is also generally much larger as compared to the hydrodynamic and thermal entrance lengths. Therefore, flow through pipes is generally assumed to be fully developed over the entire length.

9.10.2 Velocity Profile for Fully Developed, Steady, Laminar Flow

Consider a fully developed, steady, laminar flow in a pipe. Consider a fluid element of length L and radius r, as shown in Fig. 9.20.

We are interested to get the velocity profile and the pressure drop (or friction factor) during flow. This is obtained by making a force balance on a cylindrical fluid element as shown in Fig. 9.20. Forces acting on the element are: pressure forces at the ends and the shear forces on the surface; there is no change in momentum since the velocities are same at both sections 1 and 2. So, writing a force balance:

$$(p_1 - p_2) \cdot \pi \cdot r^2 = \tau \cdot (2 \cdot \pi \cdot r \cdot L) \qquad ...(a)$$

But,

$$\tau = -\mu \cdot \frac{du}{dr} \qquad ...(b)$$

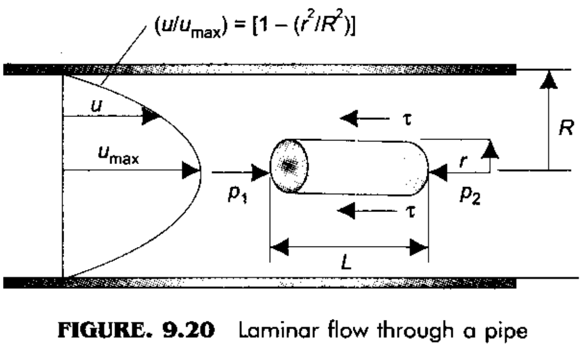


FIGURE. 9.20 Laminar flow through a pipe

(negative sign, since r is measured opposite to the direction of y).

So,

$$\frac{du}{dr} = \frac{-(p_1 - p_2)}{2 \cdot \mu \cdot L} \cdot r \qquad \text{...(c)}$$

Separating the variables and integrating,

$$\int_u^0 1 du = \frac{-(p_1 - p_2)}{2 \cdot \mu \cdot L} \cdot \int_r^R r dr \qquad \text{...(d)}$$

i.e.

$$u = \frac{1}{4 \cdot \mu} \cdot \frac{(p_1 - p_2)}{L} \cdot (R^2 - r^2) \qquad \text{...(e)}$$

This can also be written as:

$$u = \frac{-1}{4 \cdot \mu} \cdot \frac{dp}{dx} \cdot (R^2 - r^2) \qquad \text{...(9.123)}$$

since, in differential form, $\frac{dp}{dx} = \frac{-(p_1 - p_2)}{L}$

Negative sign in Eq. 9.123 indicates that pressure decreases in the flow direction. Also, note that the velocity profile is parabolic.

Now, maximum velocity occurs at $r = 0$, i.e. at the centre:

i.e.

$$u_{max} = \frac{-1}{4 \cdot \mu} \cdot \frac{dp}{dx} \cdot R^2 \qquad \text{...(9.124)}$$

Eq. 9.124 gives the maximum velocity in the pipe.

From Eqs. 9.123 and 9.124, we get:

$$\frac{u}{u_{max}} = 1 - \left(\frac{r}{R}\right)^2 \qquad \text{...(9.125)}$$

Average or mean velocity, u_m, is obtained by equating the volumetric flow to the integrated paraboloidal flow:

$$u_m \cdot \pi \cdot R^2 = \int_0^R u \cdot (2 \cdot \pi \cdot r) dr$$

i.e.

$$u_m = \frac{u_{max}}{2} = \frac{-1}{8 \cdot \mu} \cdot \frac{dp}{dx} \cdot R^2 \qquad \text{...(9.126)}$$

Now, friction factor is defined by:

$$\frac{-dp}{dx} = \frac{f}{D} \cdot \frac{\rho \cdot u_m^2}{2} \qquad \text{...(9.127)}$$

where D is the pipe diameter and $\frac{\rho \cdot u_m^2}{2}$ is the dynamic pressure.

Integrating Eq. 9.127, we get, **'Darcy – Weisbach equation'** for pressure drop:

$$\frac{\Delta p}{L} = \frac{f}{D} \cdot \frac{\rho \cdot u_m^2}{2} \qquad \text{...(9.128)}$$

where $\Delta p = p_1 - p_2$ and, $L = x_2 - x_1$

From Eqs. 9.126 and 9.127, we get:

$$f = \frac{64}{Re_D} \qquad \text{...(9.129)}$$

Eq. 9.129 gives the friction factor for laminar flow ($Re < 2000$), in a pipe flow.

Since volumetric flow rate, $Q = A.u_m$, we can write for head loss:

$$h_L = \frac{\Delta p}{\rho} = 128 \cdot \frac{Q \cdot L \cdot \mu}{\pi \cdot D^4 \cdot \rho} \quad ...(9.130)$$

Or,

$$Q = \frac{\pi}{8 \cdot \mu} \cdot \frac{R^4}{L} \cdot (p_1 - p_2) \quad ...(9.131)$$

Eq. 9.131 is known as **'Hagen – Poiseuille equation'**.

Darcy-Weisbach Eq. 9.128 is applicable to non-circular ducts also, if D is replaced by 'hydraulic diameter (D_h)', defined by:

$$D_h = \frac{4 \cdot A}{P} \quad ...(9.132)$$

where A is the area of cross-section and P is the wetted perimeter.

Values of product of friction factor and Reynolds number for two important duct configurations (viz. annular ducts and rectangular ducts) are given Tables 9.7 and 9.8 below:

TABLE 9.7 Annular ducts

Ratio of radii	f.Re
0.001	74.68
0.01	80.11
0.05	86.27
0.10	89.37
0.20	92.35
0.40	94.71
0.60	95.59
0.80	95.92
1.00	96.00

TABLE 9.8 Rectangular ducts

Ratio of sides	f.Re
0.05	89.91
0.10	84.68
0.125	82.34
0.166	78.81
0.25	72.93
0.40	65.47
0.50	62.19
0.75	57.89
1.00	56.91

9.10.3 Heat Transfer Considerations in a Pipe

Most of the practical cases of heat transfer involving a pipe flow fall under two categories:

(a) surface heat flux on the pipe is constant, e.g. when the pipe is subjected to radiation or heated electrically by winding an electric tape, or

(b) pipe surface temperature is constant, e.g. when there is condensation or boiling occurring on the surface of the pipe.

(a) Constant surface heat flux, q_s:

Let a fluid enter a pipe subjected to a constant surface heat flux q_s, with a mean inlet temperature of T_i and let the mean exit temperature of fluid be T_e. Then, the heat transfer rate can be written as:

$$Q = q_s \cdot A = m \cdot C_p \cdot (T_e - T_i) \quad ...(9.133)$$

Or, the mean exit temperature of fluid may be written as:

$$T_e = T_i + \frac{q_s \cdot A}{m \cdot C_p} \quad ...(9.134)$$

where A is the surface area of the pipe, m is the mass flow rate of fluid and C_p is its mean specific heat. Mean fluid temperature, T_m increases linearly in the flow direction. The surface temperature is determined from:

$$q_s = h \cdot (T_s - T_m) \quad ...(9.135)$$

When h is constant, for constant surface heat flux, $(T_s - T_m)$ is constant, i.e. the surface temperature also increases linearly in the flow direction. This situation is shown graphically in Fig. 9.21:

Now, we are interested to get the temperature profile and the heat transfer coefficient during flow. This is obtained by making an energy balance on a cylindrical fluid element shown in Fig. 9.22. Here, the surface heat flux along the length is constant, i.e.

$$\frac{dq_s}{dx} = 0$$

Heat flows to be considered are: conduction in and out of the element at the ends and the heat convected in and out by virtue of flow.

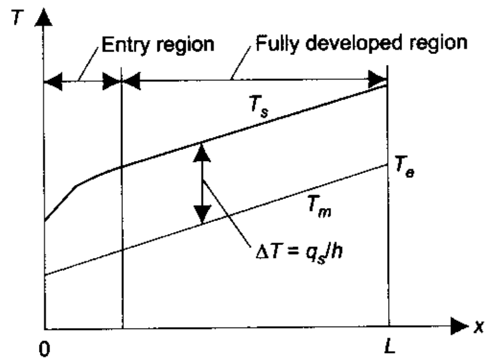


FIGURE 9.21 Tube surface and mean fluid temperatures for a pipe with constant surface heat flux

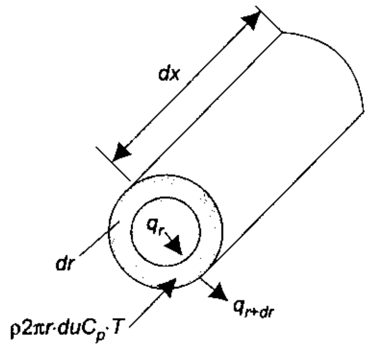


FIGURE 9.22 Control element for energy balance in pipe flow

So, writing an energy balance:

Heat flow into the element by conduction =

$$dQ_r = -k \cdot 2 \cdot \pi \cdot r \cdot dx \cdot \frac{dT}{dr}$$

Heat flow out of the element by conduction =

$$dQ_{r+dr} = -k \cdot 2 \cdot \pi \cdot (r + dr) \cdot dx \cdot \left(\frac{dT}{dr} + \frac{d^2T}{dr^2} \cdot dr \right)$$

Net heat convected out of the element is:

$$dQ_{\text{Conv}} = 2 \cdot \pi \cdot r \cdot dr \cdot \rho \cdot Cp \cdot u \cdot \frac{dT}{dx} \cdot dx$$

By energy balance:

Net energy convected out = net energy conducted in i.e. $$dQ_{Conv} = dQ_r - dQ_{r+dr}$$

Substituting for the above terms and simplifying neglecting higher order differentials,

we get: $$\frac{1}{u \cdot r} \cdot \frac{d}{dr}\left(r \cdot \frac{dT}{dr}\right) = \frac{1}{\alpha} \cdot \frac{dT}{dx} \qquad \text{...(9.136)}$$

As already discussed, with constant heat flux at the wall, average fluid temperature must increase linearly with x, so that dT/dx = constant i.e. temperature profiles will be similar at different locations along the length.

To solve Eq. 9.136, we have to insert the expression for the velocity profile given by Eq. 9.125, with the boundary conditions:

$$\frac{dT}{dr} = 0 \quad \text{at } r = 0$$

and $$k \cdot \left(\frac{dT}{dr}\right)_{r=R} = q_s = \text{constant}$$

So, Eq. 9.136 becomes:

$$\frac{d}{dr}\left(r \cdot \frac{dT}{dr}\right) = \frac{1}{\alpha} \cdot \frac{dT}{dx} \cdot u_{max} \cdot \left(1 - \frac{r^2}{R^2}\right) \cdot r$$

Integrating,

$$r \cdot \frac{dT}{dr} = \frac{1}{\alpha} \cdot \frac{dT}{dx} \cdot u_{max} \cdot \left(\frac{r^2}{2} - \frac{r^4}{4 \cdot R^2}\right) + C_1$$

Integrating again,

$$T = \frac{1}{\alpha} \cdot \frac{dT}{dx} \cdot u_{max} \cdot \left(\frac{r^2}{4} - \frac{r^4}{16 \cdot R^2}\right) + C_1 \cdot \ln(r) + C_2$$

Applying the first B.C., we get: $C_1 = 0$. Also, $T = T_c$ at $r = 0$, at centre of the pipe, i.e. $C_2 = T_C$

Therefore, temperature distribution in terms of temperature at the centre of the pipe is:

$$T - T_c = \frac{1}{\alpha} \cdot \frac{dT}{dx} \cdot \frac{u_{max} \cdot R^2}{4} \cdot \left[\left(\frac{r}{R}\right)^2 - \frac{1}{4} \cdot \left(\frac{r}{R}\right)^4\right] \qquad \text{...(9.137)}$$

Bulk temperature:

For convection heat transfer in a pipe, we have:

local heat flux, $$q = h \cdot (T_s - T_b) \qquad \text{...(9.138)}$$

where T_s is the wall temperature, and T_b the 'bulk temperature', which is an energy averaged temperature across the pipe, calculated from:

$$T_b = \frac{\int_0^R \rho \cdot 2 \cdot \pi \cdot r \cdot u \cdot C_p \cdot T dr}{\int_0^R \rho \cdot 2 \cdot \pi \cdot r \cdot u \cdot C_p \cdot dr} \qquad \text{...(9.139)}$$

Again, we have already shown that bulk temperature is a linear function of x for constant heat flux at the wall. Performing the calculation in Eq. 9.139, (using Eq. 9.137), we get:

$$T_b = T_c + \frac{7}{96} \cdot \frac{u_{max} \cdot R^2}{\alpha} \cdot \frac{dT}{dx} \qquad \text{...(9.140)}$$

And, wall (or, surface) temperature is given by:

$$T_s = T_c + \frac{3}{16} \cdot \frac{u_{max} \cdot R^2}{\alpha} \cdot \frac{dT}{dx} \qquad \text{...(from eqn. 9.137, with } r = R\text{)...(9.141)}$$

Now, the heat transfer coefficient is given by:

$$Q = h \cdot A \cdot (T_s - T_b) = k \cdot A \cdot \left(\frac{dT}{dr}\right)_{r=R}$$

i.e.
$$h = \frac{k \cdot \left(\frac{dT}{dr}\right)_{r=R}}{T_s - T_b} \qquad ...(9.142)$$

Now, the numerator in Eq. 9.142 is the temperature gradient and is given by:

$$\left(\frac{dT}{dr}\right)_{r=R} = \frac{u_{max}}{\alpha} \cdot \frac{dT}{dx} \cdot \left(\frac{r}{2} - \frac{r^3}{4 \cdot R^2}\right)_{r=R} = \frac{u_{max} \cdot R}{4 \cdot \alpha} \cdot \frac{dT}{dx} \qquad ...(9.143)$$

Substituting Eqs. 9.140, 9.141 and 9.143 in 9.142, we get

$$h = \frac{24}{11} \cdot \frac{k}{R} = \frac{48}{11} \cdot \frac{k}{D} \qquad ...(9.144)$$

Or, in terms of Nusselts number:

$$Nu_D = \frac{h \cdot D}{k} = 4.364 \qquad ...(9.145)$$

Note the interesting result that for, steady, fully developed laminar flow in a pipe whose walls are subjected to a constant heat flux, the Nusselts number is a constant = 4.364. Of course, at the entrance region, value of Nusselts number will be somewhat higher.

(b) Constant surface temperature, T_s:

Let a fluid enter a pipe whose surface is maintained at a constant temperature T_s, with a mean inlet temperature of T_i and let the mean exit temperature of fluid be T_e. Then, the mean temperature of the fluid T_m approaches the surface temperature asymptotically, as shown in Fig. 9.23.

Now, the temperature of the surface is constant and the fluid temperature varies continuously from T_i at the inlet to T_e at the exit. To determine the heat transfer rate, we have the Newton's rate equation, $Q = hA\,\Delta T_m$, where ΔT_m is a mean temperature difference between the surface and the fluid. In the chapter on heat exchangers, it will be shown that this mean temperature difference, also known as 'log mean temperature difference (LMTD)', is given as:

$$\Delta T_m = \text{LMTD} = \frac{\Delta T_e - \Delta T_i}{\ln\left(\frac{\Delta T_e}{\Delta T_i}\right)} \qquad ...(9.146)$$

where, ΔT_i and ΔT_e are the temperature differences at the inlet and outlet, as shown

Also,
$$\frac{\Delta T_e}{\Delta T_i} = \exp\left(\frac{-h \cdot A}{m \cdot C_p}\right) \qquad ...(9.147)$$

Here, m is the mass flow rate (kg/s), A is the area of heat transfer and C_p is the specific heat of the fluid.

From Eq. 9.147, one can calculate the mean fluid temperature at the exit. The term $h.A/(m.C_p)$ is known as 'Number of Transfer Units (NTU)' and is a measure of the size of the heat exchanger.

i.e.
$$\frac{\Delta T_e}{\Delta T_i} = \exp(-\text{NTU}) \qquad ...(9.148)$$

By making an analysis similar to the one as we did in the case of constant heat flux at the walls, we can show that for the case of constant wall temperature, for steady, laminar flow, the Nusselts number is a constant, given by:

$$Nu_D = \frac{h \cdot D}{k} = 3.656 \qquad ...(9.149)$$

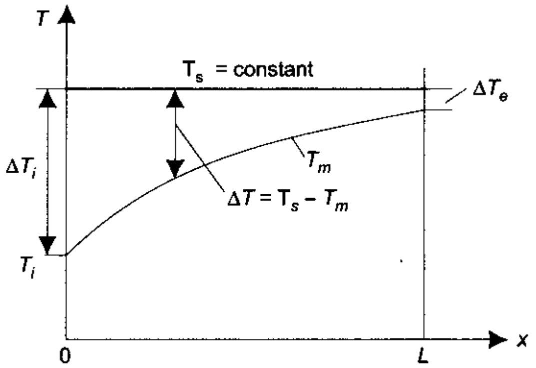


FIGURE 9.23 Variation of mean fluid temperature for a pipe with constant surface temperature

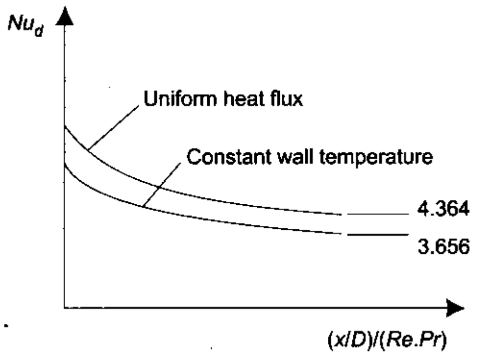


FIGURE 9.24 Variation of Nusselts number with (x/D)/(Re.Pr), for laminar flow in a pipe

Again, note that this is for fully developed flow and in the entrance region the values will be higher.

Nature of variation of Nusselts number with the dimensionless number $(x/D)/(Re.Pr)$ is shown in the following graph (Fig. 9.24).

Note that for fully developed flows, Nusselts number approaches the asymptotic values of 4.364 and 3.656 for the cases of uniform heat flux and constant wall temperature, respectively.

For short pipes (L/D is small, < 60), with constant wall temperature, fully developed velocity profile (parabolic), average Nusselt number is given by Hausen as:

$$Nu_{avg} = 3.66 + \frac{0.0668\left(\frac{D}{L}\right)\cdot Re\cdot Pr}{1+0.04\cdot\left[\left(\frac{D}{L}\right)\cdot Re\cdot Pr\right]^{\frac{2}{3}}} \qquad ...Pr > 0.7...(9.150a)$$

This equation gives the average Nusselt number over the length of tube, including the entry region. Here, $Re = (D.u_m.\rho)/\mu$. Also, in the above expression, the dimensionless group in the denominator is known as Graetz number, i.e.

$$Gz = Re\cdot Pr\cdot\frac{D}{L}$$

For oils, or other fluids in which viscosity varies with temperature considerably, the constant 0.0668 in equation 9.150a must be multiplied by $(\mu/\mu_s)^{0.14}$.

Another correlation for the above conditions is:

$$Nu_{avg} = 1.67\cdot\left(\frac{Re\cdot Pr}{\frac{L}{D}}\right)^{0.333} \qquad (for (L/D)/(Re.Pr) < 0.01,\ constant\ wall\ temperature...(9.150b))$$

In Eq. 9.150, property values are taken at mean bulk temperature. If the outlet temperature is not specified, iterative working will be required.

Another correlation to take care of the property variations is suggested by Sieder and Tate:

$$Nu_{avg} = 1.86\cdot\left(\frac{Re\cdot Pr}{\frac{L}{D}}\right)^{\frac{1}{3}}\cdot\left(\frac{\mu}{\mu_s}\right)^{0.14} \qquad ...(9.150c)$$

For short pipes (L/D is small, < 60), with constant wall temperature, velocity profile still developing, average Nusselt number is given by Hausen as:

$$Nu_{avg} = 3.66 + \frac{0.104\cdot\left(\frac{D}{L}\right)\cdot Re\cdot Pr}{1+0.016\cdot\left[\left(\frac{D}{L}\right)\cdot Re\cdot Pr\right]^{0.8}} \qquad (Pr = 0.7...(9.150d))$$

For oils, or other fluids in which viscosity varies with temperature considerably, the constant 0.104 in Eq. 9.150d must be multiplied by $(\mu/\mu_s)^{0.14}$.

For long lengths, at constant wall temperature, Nusselt number asymptotically approaches the value 3.66.

For short pipes with constant wall heat flux, with fully developed parabolic velocity profile, Hausen's correlation for local Nusselt number is:

$$Nu = 4.36 + \frac{0.023\cdot\left(\frac{D}{L}\right)\cdot Re\cdot Pr}{1+0.0012\cdot\left[\left(\frac{D}{L}\right)\cdot Re\cdot Pr\right]} \qquad (Pr > 0.7...(9.151a))$$

Another relation recommended for above conditions is:

$$Nu_{avg} = 1.30\cdot\left(\frac{Re\cdot Pr}{\frac{L}{D}}\right)^{0.33} \qquad (\textit{for } (L/D)/(Re.Pr) < 0.01, \textit{ constant wall heat flux}...(9.151b))$$

For short pipes with constant wall heat flux, with developing velocity profile, Hausen's correlation for local Nusselt number is:

$$Nu = 4.36 + \frac{0.036\cdot\left(\frac{D}{L}\right)\cdot Re\cdot Pr}{1+0.0011\cdot\left[\left(\frac{D}{L}\right)\cdot Re\cdot Pr\right]} \qquad (Pr = 0.7...(9.151c))$$

For long pipes with constant wall heat flux, average Nusselt number approaches the value 4.364, as already discussed.

9.10.4 Fully Developed Laminar Flow Inside Pipes of Non-circular Cross-sections

Nusselts number and friction factor for fully developed laminar flow inside pipes of non-circular cross-sections are given in Table 9.9. Here, Reynolds number and Nusselts number are based on the hydraulic diameter, which was defined earlier, as:

$$D_h = \frac{4\cdot A}{P} \qquad ...(9.132)$$

where A is the area of cross-section and P is the wetted perimeter.

Flow through an annulus: Practically important case is the flow through an annulus with the outer surface insulated, and the inside surface maintained at either a constant temperature or constant heat flux.

In the case of an annulus, the hydraulic diameter as given by Eq. 9.132 viz.

$D_h = (D_o - D_i)$. For fully developed laminar flow, Nusselt number varies with (D_i/D_o) as shown in Table 9.10. Here, Nu_T is the Nusselt number with the inner wall maintained at constant temperature and Nu_H is the Nusselt number with the inner surface maintained at constant heat flux. Outside surface is insulated for both the cases.

In laminar flow, surface roughness of the pipe does not have much effect on Nusselts number or friction factor.

TABLE 9.9 Nusselts number and friction factor for fully developed Laminar flow in pipes of various cross-sections

Cross-section of pipe	a/b or, θ, deg.	$Nu(T_s = const.)$	$Nu(q_s = const.)$	Friction factor, f
Circle (dia. = D)	-	3.66	4.36	64/Re
Hexagon	-	3.35	4.00	60.20/Re
Square	-	2.98	3.61	56.92/Re
Rectangle of width 'a' and height 'b'	a/b = 1	2.98	3.61	56.92/Re
	2	3.39	4.12	62.20/Re
	3	3.96	4.79	68.36/Re
	4	4.44	5.33	72.92/Re
	6	5.14	6.05	78.80/Re
	8	5.60	6.49	82.32/Re
	∞	7.54	8.24	96.00/Re
Ellipse, of major axis 'a' and minor axis 'b'	a/b = 1	3.66	4.36	64.00/Re
	2	3.74	4.56	67.28/Re
	4	3.79	4.88	72.96/Re
	8	3.72	5.09	76.60/Re
	16	3.65	5.18	78.16/Re
Triangle, with apex angle θ = (deg.)	θ = 10	1.61	2.45	50.80/Re
	30	2.26	2.91	52.28/Re
	60	2.47	3.11	53.32/Re
	90	2.34	2.98	52.60/Re
	120	2.00	2.68	50.96/Re

TABLE 9.10 Nusselt numbers for fully developed laminar flow in an annulus, insulated on the outside

D_i/D_o	0.05	0.10	0.25	0.50
Nu_T	17.46	11.56	7.37	5.74
Nu_H	17.81	11.91	8.5	6.58

9.10.5 Turbulent Flow Inside Pipes

9.10.5.1 Velocity profile and pressure drop. Experimental results of Nikuradse for turbulent flow in smooth pipes indicated a power–law form for velocity profile:

$$\frac{u}{u_{max}} = \left(\frac{y}{R}\right)^{\frac{1}{n}} \quad ...(9.152)$$

where u is the local time-average velocity, u_{max} is the time–average velocity at the centre, R is the radius of the pipe and $y = (R - r)$, is the distance from the pipe wall. Values of index n are given in Table 9.11 for different values of Reynolds numbers:

Pressure drop for turbulent flow in pipes is also given by the Darcy – Weisbach equation i.e.

$$\frac{\Delta p}{L} = \frac{f}{D} \cdot \frac{\rho \cdot u_m^2}{2} \quad ...(9.128)$$

TABLE 9.11 Values of index 'n' in Eq. 9.152 for turbulent flow in pipes

Re	n
4×10^3	6.0
2.3×10^4	6.6
1.1×10^5	7.0
1.1×10^6	8.8
2×10^6	10.0
3.2×10^6	10.0

However, friction factor f must be determined experimentally. (Note that in case of laminar flow equation for friction factor was derived analytically as $f = 64/Re$).

Average or mean velocity, u_m over the cross-section is easily calculated for the power-law profile as:

$$u_m = \frac{\int_0^R (R-r)^{\frac{1}{n}} \cdot \mu_{max}^2 \cdot 2 \cdot \pi \cdot r dr}{\int_0^R 2 \cdot \pi \cdot r dr}$$

Performing the integration we get the result as:

$$u_m = \frac{2 \cdot n^2}{(2 \cdot n + 1) \cdot (n+1)} \quad \textit{(average or mean velocity...(9.153))}$$

Friction factor 'f' for smooth pipes is given by the following empirical relations:

$$f = 0.316 \cdot Re^{-0.25} \quad (for\ 2 \times 10^4 < Re < 8 \times 10^4 ...(9.154))$$

$$f = 0.184 \cdot Re^{-0.2} \quad (for\ 10^4 < Re < 10^5 ...(9.155))$$

$$f = (0.79 \cdot \ln(Re) - 1.64)^{-2} \quad (for\ 3000 < Re < 5 < 10^6 ...(9.156))$$

Eq. 9.156 for friction factor, developed by Petukhov, covers a wide range of Reynolds numbers.

Friction factor 'f' for commercial or 'rough' pipes is given by Colebrook's formula (1939) or from the Moody's diagram. Here, surface imperfections on the internal surface extend beyond the laminar sub-layer and are characterized by a 'roughness height' 'ε' and the 'relative roughness' (ε/D) is a parameter in the Moody's diagram. See Fig. 9.25. Note that in the region of complete turbulence, friction factor is mainly dependent on the relative roughness. Values of 'ε' for commercial piping are given in Table 9.12.

Colebrook formula:

$$\frac{1}{\sqrt{f}} = 1.74 - 2 \cdot \log\left[\left(\frac{\varepsilon}{R}\right) + \frac{18.7}{Re \cdot \sqrt{f}}\right] \quad ...(9.156a)$$

Here, logarithm is to base 10. This equation is slightly difficult to calculate since f occurs on both sides of the equation and an iterative solution will be required. Instead, following formula for f is relatively easier to calculate:

$$f = \frac{1.325}{\left(\ln\left(\frac{\varepsilon}{3.7D}\right) + \frac{5.74}{Re^{0.9}}\right)^2} \quad ...(9.156b)$$

Losses in pipe fittings:

Fittings, valves, etc. are part of the piping system and they also offer resistance to flow of fluid. Losses through fittings can be quite considerable in large, industrial piping systems. Generally, head loss through a valve or fitting is expressed in the following form:

$$h_L = k_L \cdot \frac{u_m^2}{2} \quad ...(9.157)$$

Values of 'loss coefficient', k_L for some common valves and fittings are given in Table 9.13.

In practice, while calculating pressure drop in a piping system, for each valve and fitting, an 'equivalent length L_{eq}'is found out and added to the straight length of piping and then the Darcy – Weisbach equation is applied. Equivalent length for a valve or fitting is calculated from:

$$L_{eq} = \frac{k_L \cdot D}{f} \quad ...(9.158)$$

9.10.5.2 Heat transfer coefficient for turbulent flow inside pipes. Analytical treatment of turbulent flow is rather complicated as compared to that of laminar flow; therefore, empirical relations based on extensive experimental data have been suggested. Reynold's analogy between momentum and heat transfer supplies the simplest correlation:

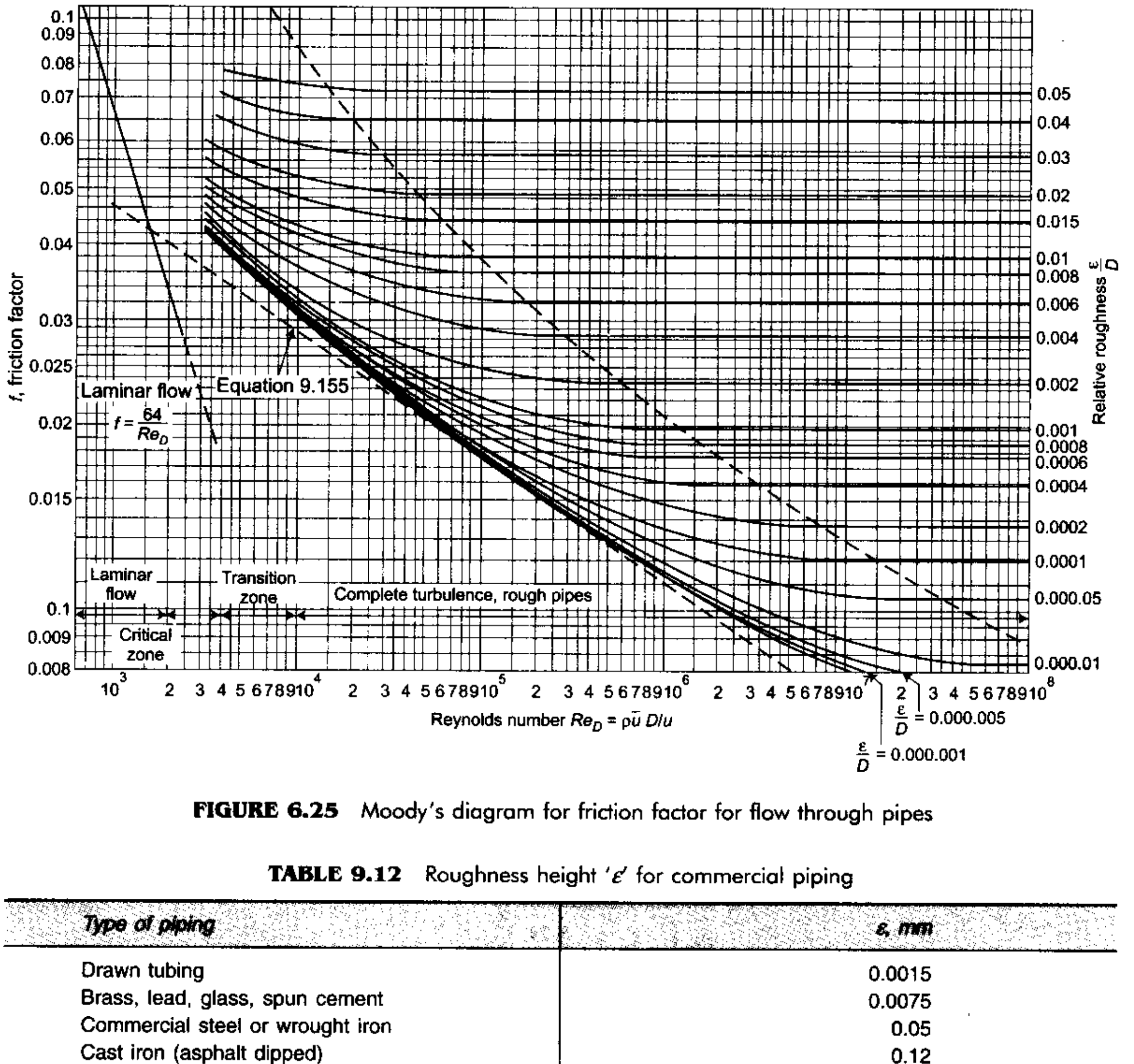


FIGURE 6.25 Moody's diagram for friction factor for flow through pipes

TABLE 9.12 Roughness height 'ε' for commercial piping

Type of piping	*ε, mm*
Drawn tubing	0.0015
Brass, lead, glass, spun cement	0.0075
Commercial steel or wrought iron	0.05
Cast iron (asphalt dipped)	0.12
Galvanized iron	0.15
Wood stave	0.2 to 1.0
Cast iron (uncoated)	0.25
Concrete	0.3 to 3.0
Riveted steel	1 to 10

Reynold's analogy between momentum and heat transfer for turbulent flow in a pipe:
In laminar flow, we have the expression for shear stress and heat transfer as follows:

$$\frac{\tau}{\rho} = \nu \cdot \frac{du}{dy} \qquad \textit{(in laminar flow)}$$

$$\frac{q}{\rho \cdot C_p} = \alpha \cdot \frac{dT}{dy} \qquad \textit{(in laminar flow)}$$

TABLE 9.13 Loss coefficient (k_L) for some common valves and fittings

Item	k_L
Angle valve, fully open	3.1 to 5.0
Ball check valve, fully open	4.5 to 7.0
Gate valve, fully open	0.19
Globe valve, fully open	10
Swing check valve, fully open	2.3 to 3.5
Regular radius elbow, screwed	0.9
Regular radius elbow, flanged	0.3
Long radius elbow, screwed	0.6
Long radius elbow, flanged	0.23
Close return bend, screwed	2.2
Flanged return bend, two elbows, regular radius	0.38
---do---, long radius	0.25
Standard Tee, screwed, flow through run	0.6
---do---flow through side	1.8

Here ν and α represent momentum and thermal diffusivity, respectively. It is a molecular phenomenon i.e. in laminar flow, momentum is transported between layers of fluid at a molecular level. However, in turbulent flow, there is an additional factor of 'eddy transport' i.e. chunks of fluid, called, 'eddies' also physically move between layers and contribute to the transport of momentum and heat. This is represented for momentum and heat transfer, respectively, as follows:

$$\frac{\tau}{\rho} = (\nu + \varepsilon_M)\cdot\frac{du}{dy} \qquad \textit{(in turbulent flow...(9.159))}$$

$$\frac{q}{\rho\cdot C_p} = (\alpha + \varepsilon_H)\cdot\frac{dT}{dy} \qquad \textit{(in turbulent flow...(9.160))}$$

Now, let us assume that momentum and heat are transported at the same rate i.e. $\varepsilon_M = \varepsilon_H$, and that the Prandtl number, $Pr = 1$. Then, dividing Eq. 9.160 by 9.159, we get:

$$\frac{q}{C_p\cdot\tau}\cdot du = dT \qquad ...(9.161)$$

Now, integrate Eq. 9.161 from the surface to the mean bulk conditions, i.e. from $T = T_s$, $u = 0$ to $T = T_b$ and $u = u_m$, assuming that q/τ is a constant at the surface = q_s/τ_s:

$$\frac{q_s}{C_p\cdot\tau_s}\cdot\int_0^{u_m} 1du = \int_{T_s}^{T_b} -1dT$$

i.e.

$$\frac{q_s\cdot u_m}{C_p\cdot\tau_s} = T_s - T_b \qquad ...(9.162)$$

Now, heat flux at the wall can be written as:

$$q_s = h\cdot(T_s - T_b) \qquad ...(9.163)$$

And, the shear stress at the wall = (shear force)/surface area

$$\tau_s = \frac{\Delta P\cdot\left(\frac{\pi\cdot D^2}{4}\right)}{\pi\cdot D\cdot L} = \frac{\Delta P}{4}\cdot\frac{D}{L}$$

where, the pressure drop = $\Delta P = f\cdot\frac{L}{D}\cdot\rho\cdot\frac{u_m^2}{2}$

So, we get:

$$\tau_s = \frac{f}{8}\cdot\rho\cdot u_m^2 \qquad ...(9.164)$$

Substituting Eqs. 9.163 and 9.164 in Eq. 9.162, we get:

$$St = \frac{h}{\rho\cdot C_p\cdot u_m} = \frac{Nu_D}{Re_d\cdot Pr} = \frac{f}{8} \qquad ...(9.165)$$

Eq. 9.165 is called 'Reynold's analogy' for fluid flow in a pipe and is valid for both laminar and turbulent flows. Note the restriction that $Pr = 1$, in Reynold's analogy i.e. it holds good for most of the gases.

For fluids with Prandtl number much different from unity, we have the **'Colburn analogy'** expressed as follows:

$$St\cdot Pr^{\frac{2}{3}} = \frac{f}{8} \qquad ...(9.166)$$

All fluid properties in Eq. 9.166 are evaluated at $(T_b + T_s)/2$, except C_p in Stanton number, which is evaluated at the bulk temperature of the fluid.

Note that by analogy between momentum and heat transfer, we get a relation between heat transfer coefficient (h) and friction coefficient (f), and by knowing any one of them, the other quantity can be calculated.

There are two more analogies, more refined than the ones already mentioned. We shall just state them:

Prandtl analogy:

$$St = \frac{\frac{f}{2}}{1 + 5\cdot\sqrt{\frac{f}{2}}\cdot(Pr - 1)} \qquad \textit{(Prandtl analogy...(9.167))}$$

Prandtl analogy reduces to Reynold's analogy when $Pr = 1$.

Von Karman analogy:

$$Nu = \frac{\left(\frac{f}{2}\right)\cdot Re\cdot Pr}{1 + 5\cdot\sqrt{\frac{f}{2}}\cdot\left[(Pr - 1) + \ln\left[1 + \frac{5}{6}\cdot(Pr - 1)\right]\right]} \qquad \textit{(Von Karman analogy...(9.168))}$$

Substituting the f relation from Eq. 9.155 in the Colburn analogy, i.e. 9.166, we get the following relation for Nusselt number for fully developed turbulent flow in smooth tubes:

$$Nu = 0.023\cdot Re^{0.8}\cdot Pr^{\frac{1}{3}} \qquad \textit{(for } 0.7 < Pr < 160,\ Re > 10{,}000\textit{...(9.169))}$$

This is known as **'Colburn equation'**.

9.10.5.3 Design equations. However, more popularly used design equation for fully developed ($L/D > 60$), turbulent flow in pipes is the **'Dittus–Boelter equation'**. (1930), given below:

$$Nu = 0.023\cdot Re^{0.8}\cdot Pr^{n} \qquad \textit{(for } 0.7 < Pr < 160,\ Re > 10{,}000\textit{...(9.170))}$$

where $n = 0.4$ for heating and $n = 0.3$ for cooling of the fluid flowing through the pipe. Here, fluid properties are evaluated at the bulk mean temperature of fluid i.e. at $T_b = (T_i + T_e)/2$, where T_i is the temperature of fluid at pipe inlet and T_e is the temperature of fluid at pipe outlet.

If the temperature difference, $(T_s - T_b)$ is significant, then variations in physical properties have to be taken into account, and in such situations correlation of Sieder and Tate (1936) is recommended:

$$Nu = 0.027\cdot Re^{0.8}\cdot Pr^{\frac{1}{3}}\cdot\left(\frac{\mu_b}{\mu_s}\right)^{0.14} \qquad \textit{(for } 0.7 < Pr < 10{,}000,\ 6000 < Re < 10^7\textit{...(9.171))}$$

A more recent relation (1970) which fits experimental results better is the following:

$$Nu = \frac{\left(\frac{f}{8}\right)\cdot Re\cdot Pr}{1.07 + 12.7\cdot\left(\frac{f}{8}\right)^{0.5}\cdot(\Pr^{0.67} - 1)}\cdot\left(\frac{\mu_b}{\mu_s}\right)^n \quad ...(9.171a)$$

where $n = 0.11$ for heating of fluids, $n = 0.25$ for cooling of fluids, $n = 0$ for constant heat flux and $\frac{\mu_b}{\mu_s}$ is to be replaced by $\frac{T_s}{T_b}$ for gases, temperature in Kelvin

Above equations can be used for the cases of heat transfer with constant wall temperature as well as uniform heat flux at the wall surface.

Also, relations for turbulent flow in circular pipes can be used for non-circular tubes as well, by replacing pipe diameter D in evaluating Reynolds number by the hydraulic diameter, $D_h = 4.A/P$.

Correlation for thermal entry region:

For the range of L/D from 10 to 400, Nusselt recommended the following relation for turbulent flow in pipes:

$$Nu = 0.036\cdot Re^{0.8}\cdot Pr^{\frac{1}{3}}\cdot\left(\frac{D}{L}\right)^{0.055} \quad (\text{for } 10 < (L/D) < 400...(9.172))$$

Here, fluid properties are evaluated at mean bulk temperature.

9.10.5.4 Turbulent flow in a long, smooth annulus. For Nusselt number, the correlations for circular pipes are used, with the hydraulic diameter taken as $D_h = D_o - D_i$.

For friction factor, following relation is proposed:

$$f_{\text{annulus}} = 0.085\cdot(Re)^{-0.25} \quad (\text{Re based on hydraulic diameter}...(9.173))$$

9.10.5.5 Correlations for liquid metals. For fully developed turbulent flow of liquid metals in smooth circular tubes $[(L/D) > 30]$, **with constant surface heat flux**, Skupinski et.al. recommend the following correlation:

$$Nu = 4.82 + 0.0185\cdot Pe^{0.827} \quad (3600 < Re < 9.05\times10^5,\ 100 < Pe < 10{,}000...(9.174))$$

Note that $Pe = Re.Pr$

More recent (1972) correlation, which fits the available data well for flow of liquid metals in pipes with constant heat flux, is due to Notter and Sleicher:

$$Nu = 6.3 + (0.0167\cdot Re^{0.85}\cdot Pr^{0.93}) \quad ...(9.175)$$

Similarly, **for constant surface temperature conditions**, for flow of liquid metals, Seban and Shimazaki recommend the following correlation for $Pe > 100$, and $[(L/D) > 30]$,

$$Nu = 5.0 + 0.025\cdot Pe^{0.8} \quad (\text{for } T_s = \text{constant } Pe > 100...(9.176))$$

9.10.5.6 Helically coiled tubes. Coiled tubes are used to enhance the heat transfer coefficient and also to accommodate a larger heat exchange surface in a given volume. Heat transfer in a coiled tube is more compared to that in a straight tube due to the contribution of secondary vortices formed as a result of centrifugal forces.

Here, we define a new dimensionless number, called **'Dean number, Dn'** as follows:

$$D_n = Re\cdot\left(\frac{D}{d_c}\right)^{\frac{1}{2}} \quad ...(9.177)$$

where D is the diameter of the tube and d_c is the diameter of the coil.

For laminar flow, following equations are recommended, depending upon the Dean number:

(a) When $Dn < 20$:

$$Nu_{\text{avg}} = 1.7\cdot(Dn^2\cdot Pr)^{\frac{1}{6}} \quad (Dn < 20,\ Dn^2\cdot Pr > 10{,}000...(9.178))$$

(b) When $20 < Dn < 100$:

$$Nu_{\text{avg}} = 0.9\cdot(Re^2\cdot Pr)^{\frac{1}{6}} \quad (20 < Dn < 100,...(9.179))$$

(c) When $Dn > 100$:

$$Nu_{avg} = 0.7 \cdot Re^{0.43} \cdot Pr^{\frac{1}{6}} \cdot \left(\frac{D}{d_c}\right)^{0.07} \qquad (100 < Dn < 830, ...(9.180))$$

All the above three Eqs. viz. 9.178, 9.179 and 9.180 are valid for $10 < Pr < 600$.

Also, for coiled tubes, there is not much difference in values of average Nusselt numbers whether the surface temperature is kept constant or the surface heat flux is maintained constant.

In laminar flow, **friction factor** for a coiled tube is obtained from:

$$f_{coiled} = \left(\frac{64}{Re}\right) \cdot \frac{21.5 \cdot Dn}{(1.56 + \log(Dn))} \cdot 5.73 \qquad (2000 > Dn > 13.5 ...(9.181))$$

Here, logarithm is to base 10.

Critical Reynolds number at which flow becomes turbulent in a coiled pipe is given as:

$$Re_{cr} = 2 \cdot \left(\frac{D}{d_c}\right)^{0.32} \cdot 10^4 \qquad (\text{for } 15 < (D/d_c) < 860 ...(9.182))$$

For values of $(D/d_c) > 860$, critical Reynolds number for a curved pipe is the same as that for a straight pipe.

For turbulent flow in forced convection in helically coiled tubes, Hausen has proposed the following correlation:

$$\frac{Nu_{a\_helical}}{Nu_{a\_straight}} = 1 + \left(\frac{21}{Re^{0.14}}\right) \cdot \left(\frac{D}{d_c}\right) \qquad ...(9.183)$$

Here, LHS is the ratio of average Nusselt numbers for helical and straight tubes, D is the diameter of the tube and d_c is the diameter of the coil.

Example 9.17. Water is heated in the annular section of a double pipe heat exchanger by electrical heating of the inner pipe. Outer pipe is insulated. Mean bulk temperature of water is 60°C. For the annulus, D_i = 2.5 cm and D_o = 5 cm. Determine the convection coefficient and pressure drop/metre length for:

(i) flow rate of 0.04 kg/s, and

(ii) flow rate of 0.5 kg/s

Solution.

Data:

$Ta := 60°C$ $\quad Di := 0.025$ m $\quad Do := 0.05$ m $\quad L := 1$ m $\quad m_1 := 0.04$ kg/s (Case (i)) $\quad m_2 := 0.5$ kg/s

First, we need the properties of water at average temperature of 60°C:

$\rho := 983.3$ kg/m³ $\quad \mu := 0.467 \times 10^{-3}$ kg/(ms) $\quad C_p := 4185$ J/(kgC) $\quad k := 0.654$ W/(mC) $\quad Pr := 2.99$

Case (i): Flow rate is 0.04 kg/s:

Since there is electrical heating of the inside tube, it is a case of constant heat flux at the wall; and, the outside surface is insulated.

Reynolds number:

To calculate Re, we need hydraulic diameter, since this is annular duct:

We have, for hydraulic diameter: $\quad D_h = \frac{4 \cdot A_c}{P}$

i.e. $\quad D_h := D_o - D_i$

i.e. $\quad D_h = 0.025$ m $\quad$ *(hydraulic diameter)*

Velocity of flow: $\quad U_1 := \frac{m_1}{\rho \cdot \left[\frac{\pi \cdot (D_o^2 - D_i^2)}{4}\right]}$ m/s

i.e. $\quad U_1 = 0.028$ m/s

Therefore, $\quad Re := \frac{D_h \cdot \rho \cdot U_1}{\mu}$

i.e. $\quad Re = 1.454 \times 10^3 < 2300$ $\quad$ ***(laminar flow)***

Heat transfer coefficient:

Therefore, this is the case of laminar flow in an annular duct, insulated from outside and subjected to constant heat flux at the inner wall. We assume fully developed flow.

Then, from Table 9.10 we get:

$$Nu_H = 6.58 \qquad (\text{for } D_i/D_o = 0.5)$$

i.e. $$\frac{h \cdot D_h}{k} = 6.58$$

i.e. $$h := \frac{6.58 \cdot k}{D_h} \ \mathrm{W/(m^2C)} \qquad (\textit{heat transfer coefficient})$$

i.e. $$h = 172.133 \ \mathbf{W/(m^2C)} \qquad (\textbf{\textit{heat transfer coefficient.}})$$

Pressure drop:

Friction factor for fully developed laminar flow in an annulus, is read from Table 9.7.

We get: $$f \cdot Re = 95.15 \qquad (\text{for ratio of radii} = 0.5)$$

Therefore, $$f := \frac{95.15}{Re} \quad \text{i.e.} \quad f = 0.065 \qquad (\textit{friction factor})$$

Therefore, pressure drop is given by:

$$\frac{\Delta p}{L} = \frac{f}{D} \cdot \frac{\rho \cdot u_m^2}{2} \qquad \ldots(9.128)$$

i.e. $$\Delta p := \frac{L}{D_h} \cdot f \cdot \frac{\rho \cdot u_1^2}{2} \qquad (Pa)$$

i.e. $$\Delta p = 0.982 \qquad (\textbf{\textit{Pa/metre length...pressure drop.}})$$

Case (ii): Flow rate is 0.5 kg/s:

Reynolds number:

Velocity of flow: $$U_2 := \frac{m_2}{\rho \cdot \left[\frac{\pi \cdot (D_o^2 - D_i^2)}{4}\right]} \ \mathrm{m/s}$$

i.e. $$U_2 = 0.345 \ \mathrm{m/s}$$

Therefore, $$Re := \frac{D_H \cdot \rho \cdot U_2}{\mu}$$

i.e. $$Re = 1.818 \times 10^4 > 2300 \qquad (\textbf{\textit{turbulent flow}})$$

Heat transfer coefficient:

Therefore, this is the case of turbulent flow in an annular duct, insulated from outside and subjected to constant heat flux at the inner wall. We assume fully developed flow. And the Dittus-Boelter correlation can be used with the hydraulic diameter substituted for tube diameter *D*.

$$Nu = 0.023 \cdot Re^{0.8} \cdot Pr^n \qquad (\text{for } 0.7 < Pr < 160,\ Re > 10{,}000 \ldots(9.170))$$

Here, $n = 0.4$, since the fluid is being heated.

i.e. $$Nu := 0.023 \cdot Re^{0.8} \cdot Pr^{0.4}$$

i.e. $$Nu = 91.117 \qquad (\textit{Nusselt number})$$

Therefore, $$h := \frac{Nu \cdot k}{D_h} \ \mathrm{W/(m^2C)} \qquad (\textit{heat transfer coefficient})$$

i.e. $$h = 2.384 \times 10^3 \ \mathbf{W/(m^2C)} \qquad (\textbf{\textit{heat transfer coefficient}})$$

Pressure drop:

Friction factor for fully developed turb. flow in an annulus, can be read from Moody's diagram, or we can use Eq. 9.154:

i.e. $$f := 0.316 \cdot Re^{-0.25} \qquad (\text{for } 2 \times 10^4 < Re < 8 \times 10^4 \ldots(9.154))$$

We get: $$f = 0.027 \qquad (\textit{friction factor})$$

Therefore, pressure drop is given by:

$$\frac{\Delta p}{L} = \frac{f}{D} \cdot \frac{\rho \cdot u_m^2}{2} \qquad \ldots(9.128)$$

i.e. $$\Delta p := \frac{L}{D_h} \cdot f \cdot \frac{\rho \cdot U_2^2}{2} \qquad (Pa)$$

i.e. $\Delta p = 63.814$ ***(Palmetre length...pressure drop.)***

Example 9.18. Water at 20°C flows through a 2.5 cm ID, 1 m long pipe, whose surface is maintained at a constant temperature of 50°C, at velocity of 5 cm/s. Determine the outlet temperature of water, assuming fully developed hydrodynamic boundary layer.

Solution.

Data:

$T_i := 20°C \quad Ts := 50°C \quad D := 0.025 \text{ m} \quad L := 1 \text{ m} \quad U := 0.05 \text{ m/s}$

We need the properties of water at mean bulk temperature. But, as yet, we do not know the exit temperature of water. So, let us assume the mean bulk temperature as 30°C and proceed with the calculations; later, we will check this assumption and refine our calculations, if required.

Properties of water at $T_b = 30°C$:

$\rho := 996.0 \text{ kg/m}^3 \quad \mu := 0.798 \times 10^{-3} \text{ kg/(ms)} \quad C_p := 4178 \text{ J/(kgC)} \quad k := 0.615 \text{ W/(mC)} \quad Pr := 5.42$

Reynolds number:

$$Re := \frac{D \cdot U \cdot \rho}{\mu}$$

i.e. $Re = 1.56 \times 10^3$ ***(< 2300...therefore, laminar flow)***

Now, $\frac{L}{D} = 40$ and $0.4 \cdot Re = 62.406$

Therefore, flow is in the entrance region

$$\frac{\frac{L}{D}}{Re \cdot Pr} = 4.73 \times 10^{-3} < 0.01$$

Therefore, we use Eq. 9.150b, viz

$$Nu_{avg} := 1.67 \cdot \left(\frac{Re \cdot Pr}{\frac{L}{D}} \right)^{0.333}$$

(for (L/D)/(Re.Pr) < 0.01, constant wall temperature...9.150b)

i.e. $Nu_{avg} = 9.931$ *(average Nusselt number)*

Therefore, $h := \frac{Nu_{avg} \cdot k}{D}$

i.e. $h = 244.293 \text{ W/(m}^2\text{C)}$ *(heat transfer coefficient)*

Now, determine the outlet temperature by an energy balance:

i.e. $$\frac{\pi \cdot D^2}{4} \cdot \rho \cdot U \cdot C_p \cdot (T_o - T_i) = \pi \cdot D \cdot L \cdot h \cdot \left(T_s - \frac{T_i + T_o}{2} \right)$$

In the above equation we have assumed that the mean temperature difference between the water stream and the surface is the difference between the surface temperature and the arithmetic mean of water temperature at inlet and exit. Strictly speaking, we should consider the LMTD; however, the assumption of arithmetic mean is good enough and the error is not much.

Let us solve this easily by Mathcad. Assume a guess value for T_o to start with, and then write the constraint after typing 'Given'. Then the command 'Find (T_o)' gives the value of T_o immediately:

$T_o := 100$ *(guess value of T_o)*

Given

$$\frac{\pi \cdot D^2}{4} \cdot \rho \cdot U \cdot C_p \cdot (T_o - T_i) = \pi \cdot D \cdot L \cdot h \cdot \left(T_s - \frac{T_i + T_o}{2} \right)$$

Find $(T_o) = 25.152$

i.e. $T_o = 25.152°C$ *(exit water temperature)*

Therefore, mean temperature of water is: (20 + 25.152)/2 = 22.5°C, whereas we had assumed a mean value of 30°C. Taking the properties of water at 22.5°C, calculations can now be repeated:

Properties of water at T_b = 22.5°C:
ρ := 997.5 kg/m³ μ := 0.95 × 10⁻³ kg/(ms) C_p := 4181 J/(kgC) k := 0.602 W/(mC) Pr := 6.575
Therefore,

$$Re := \frac{D \cdot U \cdot \rho}{\mu}$$

i.e. $Re = 1.313 \times 10^3$ (< **2300...*therefore, laminar flow***)

$$\frac{\frac{L}{D}}{Re \cdot Pr} = 4.635 \times 10^{-3} < 0.01$$

Therefore, we use Eq. 9.150b i.e.

$$Nu_{avg} := 1.67 \cdot \left(\frac{Re \cdot Pr}{\frac{L}{D}} \right)^{0.333}$$ *(for (L/D)/(Re.Pr) < 0.01, constant wall temperature....9.150b)*

i.e. $Nu_{avg} = 9.998$ *(average Nusselt number)*

Therefore, $h := \frac{Nu_{avg} \cdot k}{D}$

i.e. $h = 240.754$ W/(m²C) *(heat transfer coefficient)*

Now, determine the outlet temperature by an energy balance, and using Solve block of Mathcad:

$T_o := 100$ *(guess value of T_o)*

Given

$$\frac{\pi \cdot D^2}{4} \cdot \rho \cdot U \cdot C_p \cdot (T_o - T_i) = \pi \cdot D \cdot L \cdot h \cdot \left(T_s - \frac{T_i + T_o}{2} \right)$$

Find $(To) = 25.073$

i.e. $T_o = 25.073$°C ***(exit water temperature.)***

Therefore, T_b = (20 + 25.073)/2 = 22.573°C, which is very close to T_b = 22.5°C at which properties of water were taken. So, T_o = 25.073°C **...(Ans).**

Example 9.19. Air at 1 bar and 20°C flows through a 6 mm ID, 1 m long smooth pipe, whose surface is maintained at a constant heat flux, with velocity of 3 m/s. Determine the heat transfer coefficient if the exit bulk temperature of air is 80°C. Also determine the exit wall temperature and the value of h at the exit.

Solution.

Data:

T_i := 20°C T_o := 80°C D := 0.006 m L := 1 m U := 3.0 m/s

Therefore, mean bulk temperature is (20 + 80)/2 = 50°C

i.e. T_b = 50°C *(mean bulk temperature of air)*

Properties of air at T_b = 50°C:
ρ := 1.093 kg/m³ μ := 19.61 × 10⁻⁶ kg/(ms) C_p := 1005 J/(kgC) k := 0.02826 W/(mC) Pr := 0.698
Reynolds number:

$$Re := \frac{D \cdot U \cdot \rho}{\mu}$$

i.e. $Re = 1.003 \times 10^3$ (< **2300...*therefore, laminar flow***)

Since the tuube length is short, entrance effect must be considered.

We have: $\frac{L}{D} = 166.667$

and, $$\frac{\frac{L}{D}}{Re \cdot Pr} = 0.238$$ > 0.01

Nusselt number:

Therefore, we shall use following equation assuming developing velocity profile:

$$Nu := 4.36 + \frac{0.036 \cdot \left(\frac{D}{L}\right) \cdot Re \cdot Pr}{1 + 0.0011 \cdot \left[\left(\frac{D}{L}\right) \cdot Re \cdot Pr\right]}$$ (*Pr* = 0.7...(9.151c))

i.e. $Nu = 4.511$ (*Nusselt number*)

Heat transfer coefficient:

Therefore, $h := \frac{Nu \cdot k}{D}$ W/(m²C) (*heat transfer coefficient*)

i.e. $h = 21.245$ W/(m²C) (*heat transfer coefficient*)

Exit wall temperature:

Since the wall heat flux is constant, we have the relation for *h*:

$$h = \frac{q_w}{(T_s - T_b)}$$ ...(a)

Also,

Mass flow rate: $m := \rho \cdot \left(\frac{\pi \cdot D^2}{4}\right) \cdot U$

i.e. $m = 9.27 \times 10^{-5}$ kg/s

and, $Q := m \cdot C_p \cdot (T_o - T_i)$ W (*total heat transfer rate*)

i.e. $Q = 5.591$ W (*total heat transfer rate*)

But, $Q = q_w \cdot \pi \cdot D \cdot L$ (where q_w is the constant surface heat flux)

Therefore, $q_w := \frac{Q}{\pi \cdot D \cdot L}$ W/m² (*surface heat flux*)

i.e. $q_w = 296.586$ W/m² (*surface heat flux*)

Therefore, from Eq. a:

$$T_{w\_exit} := T_o + \frac{q_w}{h} \,°C$$ (*surface temperature at exit*)

i.e. $T_{w\_exit} = 93.96°C$ (*surface temperature at exit.*)

Example 9.20. Water (under pressure) is heated in an economiser from a temperature of 30°C to 150°C. Tube wall is maintained at a constant temperature of 350°C. If the water flows at a velocity of 1.5 m/s and the tube diameter is 50 mm, determine the length of tube required.

Solution.

Data:

$T_i := 30°C$ $T_o := 150°C$ $T_s := 350°C$ $D := 0.05$ m $U := 1.5$ m/s

Therefore, mean bulk temperature is (30 + 150)/2 = 90°C

i.e $T_b := 90°C$ ...*mean bulk temperature of water*

Properties of water at $T_b = 90°C$:

$\rho := 965.3$ kg/m³ $\mu := 0.315 \times 10^{-3}$ kg/(m/s) $C_p := 4206$ J/(kgC) $k := 0.675$ W/(mC)

$Pr := 1.96$

Reynolds number:

$$Re := \frac{D \cdot U \cdot \rho}{\mu}$$

i.e. $Re = 2.298 \times 10^5$ (**> 2300...*therefore, turbulent flow***)

Heat transfer coefficient

Using more recent correlation,

$$Nu = \frac{\left(\frac{f}{8}\right) \cdot Re \cdot Pr}{1.07 + 12.7 \cdot \left(\frac{f}{8}\right)^{0.5} \cdot (Pr^{0.67} - 1)} \cdot \left(\frac{\mu_b}{\mu_s}\right)^n$$ ...(9.171a)

where $n = 0.11$ for heating of fluids, $n = 0.25$ for cooling of fluids, $n = 0$ for constant heat flux and $\frac{\mu_b}{\mu_s}$ is to be replaced by $\frac{T_s}{T_b}$ for gases, temperature in Kelvin

We have:

$$f := (0.79 \cdot \ln(Re) - 1.64)^{-2} \qquad \text{...(9.156)}$$

i.e. $f = 0.015$

and, dynamic viscosity of water at wall temperature of 350°C is:

$$\mu_s = 0.065 \times 10^{-3} \text{ kg/(m/s)}$$

Therefore,

Then,

$$Nu := \frac{\left(\frac{f}{8}\right) \cdot Re \cdot Pr}{1.07 + 12.7 \cdot \left(\frac{f}{8}\right)^{0.5} \cdot (Pr^{0.67} - 1)} \left(\frac{\mu}{\mu_s}\right)^{0.11}$$

i.e. $Nu = 734.689$

Therefore,

$$h := \frac{Nu \cdot k}{D}$$

i.e. $h = 9.918 \times 10^3$ W/(m^{2}C) *(heat transfer coefficient)*

Length of tube required:

Water temperature varies continuously from 30°C at inlet to 150°C at exit, tube surface temperature remainin constant at 350°C. So, mean temperature difference in Newton's equation is LMTD, to be very accurate.

$$\text{LMTD} := \frac{(T_s - T_i) - (T_s - T_o)}{\ln\left[\frac{(T_s - T_i)}{(T_s - T_o)}\right]}$$

i.e. LMTD = 255.317°C *(log mean temperature difference)*

Applying energy balance:

$$\rho \cdot \left(\frac{\pi \cdot D^2}{4}\right) \cdot U \cdot C_p \cdot (T_o - T_i) = h \cdot (\pi \cdot D \cdot L) \cdot \text{LMTD}$$

Therefore,

$$L := \frac{\rho \cdot \left(\frac{\pi \cdot D^2}{4}\right) \cdot U \cdot C_p \cdot (T_o - T_i)}{h \cdot (\pi \cdot D) \cdot \text{LMTD}}$$

i.e. L = 3.607 m ***(length of tube required.)***

Note: We could have taken the mean temperature difference as the difference between surface temperature and the arithmetic mean between inlet and exit of water i.e. $\Delta T = 350 - 90 = 260$ whereas LMTD was 255.7°C. Then, L would have been 3.542 m, not much different from 3.6 m; however, using LMTD is accurate method.

Alternatively, if we had used **Dittus–Boelter equation,** viz.

$$Nu := 0.023 \cdot Re^{0.8} \cdot Pr^{0.4} \qquad (Pr^{0.4} \textit{ since fluid is being heated})\text{...(9.170)}$$

i.e. $Nu = 585.815$

and

$$h := \frac{Nu \cdot k}{D}$$

i.e. $h = 7.909 \times 10^3$ W/(m^{2}C) *...heat transfer coefficient.*

And, using LMTD we would have got L = 4.524 m

Example 9.21. Sodium potassium alloy (25:75), flowing at a rate of 3 kg/s, is heated in a tube of 5 cm ID from 200°C to 400°C. Tube surface is maintained at constant heat flux and the temperature difference between the tube surface and the mean bulk temperature of fluid is 40°C. Determine the heat transfer coefficient, heat flux at the surface and length of tube required.

Solution.

Data:

$T_i := 200°C$ $\quad T_o := 400°C$ $\quad \Delta T := 40°C$ $\quad D := 0.05$ m $\quad m := 3.0$ kg/s

Therefore mean bulk temperature is (200 + 400)/2 = 300°C

i.e. $T_b = 300°C$ *...mean bulk temperature of Na–K alloy*

Properties of Na–K alloy at $T_b = 300°C$:

$\rho := 799$ kg/m^3 $\quad \nu := 0.366 \times 10^{-6}$ m^2/s $\quad C_p := 1038.3$ J/(kgC) $\quad k := 22.68$ W/(mC) $\quad Pr := 0.0134$

Reynolds number:

$$G := \frac{m}{\left(\frac{\pi \cdot D^2}{4}\right)} \text{ kg/(sm}^2\text{)}$$ *(mass velocity)*

i.e. $G = 1.528 \times 10^3$ kg/(sm^2) *(mass velocity)*

$$Re := \frac{G \cdot D}{\nu \cdot \rho}$$ *(Reynolds number)*

i.e. $Re = 2.612 \times 10^5$ *(> 2300...therefore, turbulent flow)*

Heat transfer coefficient

Using the recent correlation of Notter and Sleicher, fo constant heat flux conditions

$$Nu := 6.3 + (0.0167 \cdot Re^{0.85} \cdot Pr^{0.93})$$ ...(9.175)

i.e. $Nu = 18.473$ *(Nusselt number)*

Therefore, $$h := \frac{Nu \cdot k}{D} \text{ W/(m}^2\text{C)}$$ *(heat transfer coefficient)*

i.e. $h = 8.379 \times 10^3$ W/(m^{2}C) *...heat transfer coefficient*

Heat flux at surface:

Now, heat flux is determined from its definition:

$$h = \frac{q_s}{\Delta T}$$ *(where q_s is the surface heat flux and ΔT is the temperature difference between surface and the bulk temperature = 40°C, a constant for constant heat flux conditions.)*

i.e. $q_s := h \cdot \Delta T$ W/m^2 *(surface heat flux)*

i.e. $q_s = 3.352 \times 10^5$ W/m^2 *(surface heat flux.)*

Length of tube required:

This is obtained by a heat balance:

$$q_s \cdot (\pi \cdot D \cdot L) = m \cdot Cp \cdot (T_o - T_i)$$

i.e. $$L := \frac{m \cdot C_p \cdot (T_o - T_i)}{q_s \cdot \pi \cdot D}$$

i.e. $L = 11.833$ m *(length of tube required.)*

Alternatively:

$$h \cdot (\pi \cdot D \cdot L) \cdot \Delta T = m \cdot C_p \cdot (T_o - T_i)$$

$$L := \frac{m \cdot C_p \cdot (T_o - T_i)}{h \cdot (\pi \cdot D) \cdot \Delta T}$$

i.e. $L = 11.833$ m *(same as earlier.)*

Also, if we use Eq. 9.174 to determine heat transfer coefficient:

$$Nu := 4.82 + 0.0185 \cdot (Re \cdot Pr)^{0.827}$$ $(3600 < Re < 9.05 \times 10^5,\ 100 < Pe < 10{,}000$...(9.174))

i.e. $Nu = 20.602$

Therefore, $$h := \frac{Nu \cdot k}{D} \text{ W/(m}^2\text{C)}$$ *(heat transfer coefficient)*

i.e. $h = 9.345 \times 10^3$ W/(m^{2}C) *(heat transfer coefficient)*

Compare this value of *h* with that obtained earlier using Eq. 9.175.

And,

$$L := \frac{m \cdot C_p \cdot (T_o - T_i)}{h \cdot (\pi \cdot D) \cdot \Delta T}$$

i.e. $L = 10.61$ m *(length of tube required)*

Example 9.22. 180 kg/h of air at one atm. pressure is cooled from 100°C to 20°C while passing through a 3 cm ID pipe coil bent into a helix of 0.7 m diameter. Calculate the air side heat transfer coefficient.

Solution.

Data:

$T_i := 200°C$ $T_o := 20°C$ $D := 0.03$ m $d_c := 0.7$ m $m := \frac{180}{3600}$ kg/s i.e. $m = 0.05$ kg/s

Therefore mean bulk temperature is (100 + 20)/2 = 60°C

i.e. $T_b := 60°C$ *(mean bulk temperature of air)*

Properties of air at $T_b = 60°C$:

$\rho := 1.06$ kg/m^3 $\mu := 20.10 \times 10^{-6}$ kg/(ms) $C_p := 1005$ J/(KgC) $k := 0.02896$ W/(mC) $Pr := 0.696$

Reynolds number:

$$G := \frac{m}{\left(\frac{\pi \cdot D^2}{4}\right)} \text{ kg/(sm}^2) \qquad \textit{(mass velocity)}$$

i.e. $G = 70.736$ kg/(sm^2) *(mass velocity)*

$$Re := \frac{G \cdot D}{\mu} \qquad \textit{(Reynolds number)}$$

i.e. $Re = 1.056 \times 10^5$ ...> 2300...*therefore, turbulent flow*

Nusselt number for straight tube:

Using the Dittus-Boelter equation for turbulent flow:

$Nu = 0.023 \cdot Re^{0.8} \cdot Pr^n$ *(for 0.7 < Pr < 160, Re > 10,000...(9.170))*

i.e. $Nu := 0.023 \cdot Re^{0.8} \cdot Pr^{0.3}$ *(n = 0.3 since air is being cooled.)*

i.e. $Nu = 215.457$ *(Nusselt number...for straight tube)*

Nusselt number for helical coil:

We have:

$$\frac{Nu_{a\_helical}}{Nu_{a\_straight}} = 1 + \left(\frac{21}{Re^{0.14}}\right) \cdot \left(\frac{D}{d_c}\right) \qquad ...(9.183)$$

Therefore,

$$Nu_{a\_helical} := Nu \cdot \left[1 + \left(\frac{21}{Re^{0.14}}\right) \cdot \left(\frac{D}{d_c}\right)\right]$$

i.e. $Nu_{a\_helical} = 253.855$

Heat transfer coefficient:

Therefore, $$h := \frac{Nu_{a\_helical} \cdot k}{D} \text{ W/(m}^2\text{C)} \qquad \textit{(heat transfer coefficient)}$$

i.e. $h = 245.054$ **W/(m^{2}C)** ***(heat transfer coefficient.)***

Example 9.23. In a long annulus (3.125 cm ID, 5 cm OD), air is heated by maintaining the temperature of outer surface of the inner tube at 50°C. The air enters at 16°C and leaves at 32°C and its flow velocity is 30 m/s. Estimate the heat transfer coefficient between the air and the inner tube. Use Dittus - Boelter equation, viz.

$Nu_D = 0.023.(Re_D)^{0.8}.Pr^{0.4}$; Average properties of air at 24°C are:

$\rho = 1.614$ kg/m^3, $C_p = 1007$ J/(kgC), $k = 0.0263$ W/(mC), $Pr = 0.7$ $\nu = 15.9 \times 10^{-6}$ m^2/s (M.U. 1999)

Solution.

Data:

$T_i := 16°C$ $T_o := 32°C$ $T_s := 50°C$ $D_i := 0.03125$ m $D_o := 0.05$ m $L := 1$ m $U := 30$ m/s

Reynolds number:

To calculate Re, we need hydraulic diameter, since this is annular duct:

We have, for hydraulic diameter: $D_h = \frac{4 \cdot A_c}{P}$

i.e. $D_h := D_o - D_i$

i.e. $D_h = 0.019$ m *(hydraulic diameter)*

Therefore, $Re := \frac{D_h \cdot U}{\nu}$

i.e. $Re = 3.538 \times 10^4$ **(> 2300...*turbulent flow*)**

Heat transfer coefficient:

We have:

$Nu := 0.023 \cdot Re^{0.8} \cdot Pr^{0.4}$ *(Dittus–Boelter equation n = 0.4, since air is being heated)*

i.e. $Nu = 86.846$ *(Nusselt number)*

Therefore, $h := \frac{Nu \cdot k}{D_h}$

i.e. $h = 121.815$ W/(m^{2}C) *(heat transfer coefficient)*

Also, calculate the pressure drop per metre length:

Friction factor:

We have, from Eq. 9.155:

$f := 0.184 \cdot Re^{-0.2}$ *(for $10^4 < Re < 10^5$...(9.155))*

i.e. $f = 0.023$ *(friction factor)*

Pressure drop:

Therefore, $\Delta P := f \cdot \frac{L}{D_h} \cdot \frac{\rho \cdot U^2}{2}$ Pa (= N/m^2) *(pressure drop per meter length)*

i.e. $\Delta P = 877.38$ **Pa** ***(pressure drop per meter length.)***

Example 9.24. Water at 20°C flows flows through a tube, 4 cm diameter 9 m length, tube surface being maintained at 90°C. Temperature of water increases from 20°C to 60°C. Find the mass flow rate. Use Dittus–Boelter equation, viz. $Nu_D = 0.023.(Re_D)^{0.8}.Pr^{0.4}$; Take properties of water at mean bulk temperature of 40°C as:

$\rho = 993$ kg/m^3, $C_p = 4170$ J/kgC), $k = 0.64$ W/(mC), $\nu = 0.65 \times 10^{-6}$ m^{2}s (M.U., 1996)

Solution.

Data:

$T_i := 20°C$ $T_o := 60°C$ $T_s := 90°C$ $D := 0.04$ m $L := 9$ m $\mu := \nu \cdot \rho$ i.e. $\mu = 6.455 \times 10^{-4}$ kg/(ms)

Therefore, $Pr := \frac{\mu \cdot C_p}{k}$ i.e. $Pr = 4.206$

Now, from Dittus–Boelter equation we get Nusselt number, hence the heat transfer coefficient h; then writing a heat balance:

Let m be the mass flow rate (kg/s) of water.

Heat gained by water = heat transferred between the pipe surface and the bulk of water

i.e. $m \cdot C_p \cdot (T_o - T_i) = h \cdot A_s \cdot \text{LMTD}$

i.e. $$m \cdot C_p \cdot (T_o - T_i) = \left[\frac{k}{D} \cdot 0.023 \left(\frac{m \cdot D}{A_c \cdot \mu} \right)^{0.8} \cdot Pr^{0.4} \cdot A_s \cdot \text{LMTD} \right]$$

where $A_c := \frac{\pi \cdot D^2}{4}$ i.e. $A_c = 1.257 \times 10^{-3}$ m^2 *(area of cross-section)*

$A_s := \pi \cdot D \cdot L$ i.e. $A_s = 1.131$ m^2 *(surface area of heat transfer)*

$$\text{LMTD} := \frac{(T_s - T_i) - (T_s - T_o)}{\ln\left[\frac{(T_s - T_i)}{(T_s - T_o)}\right]}$$ i.e. LMTD = 47.209°C

Therefore, $$m := \left[\frac{\left[\frac{k}{D}\cdot 0.023\left(\frac{D}{A_c\cdot\mu}\right)^{0.8}\cdot Pr^{0.4}\cdot A_s\cdot \text{LMTD}\right]}{[C_p\cdot(T_o - T_i)]}\right]^{\frac{1}{0.2}}$$

i.e. $m = 2.373$ **kg/s** *(mass flow rate of water)*

9.11 Summary of Basic Equations for Forced Convection

Geometry/details	Correlation	Restrictions
Flat Plate, laminar flow:		
Hydrodynamic boundary layer thickness	$\delta_{lam} = \frac{5\cdot x}{(Re_x)^{0.5}}$	$Re < 5 \times 10^5$
Local friction coefficient	$C_{fx} = \frac{\tau}{\left(\frac{\rho\cdot U^2}{2}\right)} = \frac{66.4}{\sqrt{Re_x}}$	$Re < 5 \times 10^5$
Local Nusselt number	$\frac{h\cdot x}{k} = Nu_x = 0.332\cdot\sqrt{Re_x}\cdot Pr^{0.333}$	$Re < 5 \times 10^5$, $Pr > 0.5$
Average Frction coefficient	$C_{fa} = \frac{1}{L}\cdot\int_0^L C_{fx}\,dx = \frac{1.328}{\sqrt{Re_L}}$	$Re < 5 \times 10^5$
Average Nusselt number	$Nu_a = 0.664\cdot\sqrt{Re_L}\cdot Pr^{0.333}$	$Re < 5 \times 10^5$, $Pr > 0.5$
Local Nusselt number for liquid metals	$Nu_x = 0.565\cdot Pe_x^{0.5}$ ...(Pr <.005)	$Re < 5 \times 10^5$ **$Pe = Re. Pr$**
Flat Plate, turbulent flow:		
Hydrody. b.l. thickness	$\delta_{turb} = \frac{0.371\cdot x}{(Re_x)^{0.2}}$	$Re_x > 5 \times 10^5$
Local friction coefficient	$C_{fx} = 0.0576\cdot Re_x^{\frac{-1}{5}}$	$Re_x > 5 \times 10^5$, $Pr > 0.5$
Local Nusselt number	$Nu_x = \frac{h_x\cdot x}{k} = 0.288\cdot Re_x^{0.8}\cdot Pr^{\frac{1}{3}}$	$Re_x > 5 \times 10^5$, $Pr > 0.5$
Average Friction coefficient	$C_{fa} = 0.072\cdot Re_L^{\frac{-1}{5}}$	$5 \times 10^5 < Re_L < 10^7$
Average Friction coefficient	$C_{fa} = \frac{0.455}{(\log(Re_L))^{2.58}}$	$10^7 < Re_L < 10^9$
Flat Plate, mixed boundary layer:		
Average Friction coefficient	$C_{fa} = \frac{0.074}{Re_L^{\frac{1}{5}}} - \frac{1742}{Re_L}$	$5 \times 10^5 < Re_L < 10^7$ $Re_{x,c} = 5 \times 10^5$

Contd.

Contd.

Average Nusselt number	$Nu_{avg} = \frac{h \cdot L}{k} = \left(0.036 \cdot Re_L^{\frac{4}{5}} - 836\right) \cdot Pr^{\frac{1}{3}}$	$0.6 < Pr < 60$, $5 \times 10^5 < Re_L < 10^7$
Cylinder in cross flow: Average Nusselt number	$Nu_{cyl} = \frac{h \cdot D}{k} = 0.3 + \frac{0.62 \cdot Re^{\frac{1}{2}} \cdot Pr^{\frac{1}{3}}}{\left[1 + \left(\frac{0.4}{Pr}\right)^{\frac{2}{3}}\right]^{\frac{1}{4}}} \cdot \left[1 + \left(\frac{Re}{28200}\right)^{\frac{5}{8}}\right]^{\frac{4}{5}}$	$100 < Re < 10^7$; $Re.Pr > 0.2$
Cylinder in liquid metal cross flow	$Nu_{cyl} = 1.125 \cdot (Re \cdot Pr)^{0.413}$ ...*for* $1 < Re.Pr < 100$	
Flow across a sphere: Comprehensive equation of Whitaker. Average Nusselt number	$Nu_{sph} = 2 + \left(0.4 \cdot Re^{\frac{1}{2}} + 0.06 \cdot Re^{\frac{2}{3}}\right) \cdot Pr^{0.4} \cdot \left(\frac{\mu_a}{\mu_w}\right)$	For gases & liquids. $3.5 < Re < 7.6.10^4$, $0.71 < Pr < 380$, $1 < \mu/\mu_s < 3.2$
Falling drop: **Average Nusselt no.**	$Nu_{avg} = 2 + 0.6 \cdot Re^{\frac{1}{2}} \cdot Pr^{\frac{1}{3}}$	
Flow across Tube bank: **Turbulent flow** ($Re_D > 2 \times 10^5$)	$Nu_a = 0.021 \cdot Re_D^{0.84} \cdot Pr^{0.36} \cdot (Pr/Pr_w)^{0.25}$ ...*for In-line tubes,* $Nu_a = 0.022 \cdot Re_D^{0.84} \cdot Pr^{0.36} \cdot \left(\frac{Pr}{Pr_w}\right)^{0.25}$ ...*for staggered tubes,* $Pr > 1$ $Nu_a = 0.019 \cdot Re_D^{0.84}$ ...*for staggered tubes,* $Pr = 0.7$	$N > 20$, and $0.7 < Pr < 500$, $1000 < Re_{D\_max} < 2 \times 10^6$
Flow across Tube banks: Pressure drop	$\Delta p = \frac{2 \cdot f \cdot G_{max}^2 \cdot N}{\rho} \cdot \left(\frac{\mu_w}{\mu_b}\right)^{0.14}$ Pa	$G_{max} = \rho \cdot u_{max}$; N = No. of transverse rows
Friction factor in Eq. 9.118	$f = \left[0.25 + \frac{0.118}{\left[\frac{(S_T - D)}{D}\right]^{1.08}}\right] \cdot Re_D^{-0.16}$ ...*for staggered tubes.*	
Friction factor in Eq. 9.118	$f = \left[0.044 + \frac{0.08 \cdot \left(\frac{S_L}{D}\right)}{\left[\frac{(S_T - D)}{D}\right]^{0.43 + 1.13 \cdot \frac{D}{S_L}}}\right] \cdot Re_D^{-0.15}$ ...*for in-line tubes*	

Contd.

Contd.

Flow through packed beds: **Heat transfer between gas and packings**	$\frac{h_a \cdot D_p}{k} = \frac{1-\varepsilon}{\varepsilon} \cdot \left(0.5 \cdot Re_{Dp}^{\frac{1}{2}} + 0.2 \cdot Re_{Dp}^{\frac{2}{3}}\right) \cdot Pr^{\frac{1}{3}}$	$20 < Re_{Dp} < 10{,}000$, $0.34 < \varepsilon < 0.78$. See text for definition of Re_{Dp} and ε
Flow through packed beds: **Heat transfer between walls of bed and gas**	$\frac{h_a \cdot D_p}{k} = 2.58 \cdot Re_{Dp}^{\frac{1}{3}} \cdot Pr^{\frac{1}{3}} + 0.094 \cdot Re_{Dp}^{0.8} \cdot Pr^{0.4}$	For particles like cylinders, see text for definition of Re_{Dp}
Flow through packed beds: **Heat transfer between walls of bed and gas**	$\frac{h_a \cdot D_p}{k} = 0.203 \cdot Re_{Dp}^{\frac{1}{3}} \cdot Pr^{\frac{1}{3}} + 0.220 \cdot Re_{Dp}^{0.8} \cdot Pr^{0.4}$	For particles like spheres, $40 < Re_{Dp} < 2000$ see text for definition of Re_{Dp}
Flow inside tubes: Hydrodynamic and thermal entry lengths	$L_{h\_lam} = 0.05 \cdot Re \cdot D$ $L_{t\_lam} = 0.05 \cdot Re \cdot Pr \cdot D$ $L_{h\_turb} = L_{t\_turb} = 10 \cdot D$	$Re < 2300$...laminar $Re > 4000$...turbulent
Darcy–Weisbach equation for pressure drop	$\frac{\Delta p}{L} = \frac{f}{D} \cdot \frac{\rho \cdot u_m^2}{2}$	
Friction factor	$f = \frac{64}{Re_D}$	Laminar flow in tubes
Flow inside tubes: Nusselt no. for fully developed laminar flow, constant wall heat flux	$Nu_D = \frac{h \cdot D}{k} = 4.364$	$Pr > 0.6$
Flow inside tubes: Nusselt no. for fully developed laminar flow, constant wall temperature	$Nu_D = \frac{h \cdot D}{k} = 3.66$	$Pr > 0.6$
Flow inside short tubes: Nusselt no. for fully developed velocity profile, laminar flow, constant wall temperature	$Nu_{avg} = 3.66 + \frac{0.0668\left(\frac{D}{L}\right) \cdot Re \cdot Pr}{1 + 0.04 \cdot \left[\left(\frac{D}{L}\right) \cdot Re \cdot Pr\right]^{\frac{2}{3}}}$ ...$Pr > 0.7$	$L/D < 60$
Flow inside short tubes: Nusselt no. for fully developed velocity profile, laminar flow, constant wall temperature..Sieder & Tate relation.	$Nu_{avg} = 1.86 \cdot \left(\frac{Re \cdot Pr}{\frac{L}{D}}\right)^{\frac{1}{3}} \cdot \left(\frac{\mu}{\mu_s}\right)^{0.14}$	$\left(\frac{Re \cdot Pr}{\frac{L}{D}}\right)^{\frac{1}{3}} \cdot \left(\frac{\mu}{\mu_s}\right)^{0.14} \le 2$

Contd.

Contd.

		$0.48 < Pr < 16{,}700$ $0.0044 < (\mu/\mu_s) < 9.75$
Flow inside short tubes: Local Nusselt no. for fully developed velocity profile, laminar flow, constant wall heat flux.	$Nu = 4.36 + \dfrac{0.023\cdot\left(\frac{D}{L}\right)\cdot Re\cdot Pr}{1+0.0012\cdot\left[\left(\frac{D}{L}\right)\cdot Re\cdot Pr\right]}$ ...$Pr > 0.7$	
Flow inside tubes: Friction factor for smooth pipes	$f = 0.316\cdot Re^{-0.25}$ ...*for* $2\times10^4 < Re < 8 < 10^4$ $f = 0.184\cdot Re^{-0.2}$ ...*for* $10^4 < Re < 10^5$. $f = (0.79\cdot\ln(Re) - 1.64)^{-2}$ ...*for* $3000 < Re < 5\times10^6$	
Flow inside tubes: Friction factor for rough pipe	$f = \dfrac{1.325}{\left[\ln\left(\frac{\varepsilon}{3.7D}\right)+\frac{5.74}{Re^{0.9}}\right]^2}$	Relative roughness, (ε/D) is known
Reynold's analogy	$St = \dfrac{h}{\rho\cdot C_p\cdot u_m} = \dfrac{Nu_D}{Re_d\cdot Pr} = \dfrac{f}{8}$	
Colburn analogy	$St\cdot Pr^{\frac{2}{3}} = \dfrac{f}{8}$	
Flow inside tubes: **Turbulent flow:** Nusselt number	$Nu = 0.023\cdot Re^{0.8}\cdot Pr^n$ ...*for* $0.7 < Pr < 160$, $Re > 10{,}000$ $n = 0.4$ when fluid is being heated, and $n = 0.3$ when fluid is being cooled	Dittus–Boelter equation $0.6 < Pr < 160$ $Re > 10{,}000$ $L/D > 10$
Flow inside tubes: **Turbent flow:** Nusselt number, when there is property variation	$Nu = 0.027\cdot Re^{0.8}\cdot Pr^{\frac{1}{3}}\cdot\left(\dfrac{\mu_b}{\mu_s}\right)^{0.14}$	Sieder–Tate eqn. $0.7 < Pr < 16{,}700$, $6000 < Re < 10^7$
Flow inside tubes: **Turbulent flow:** Nusselt number	$Nu = \dfrac{\left(\frac{f}{8}\right)\cdot Re\cdot Pr}{1.07 + 12.7\cdot\left(\frac{f}{8}\right)^{0.5}\cdot(Pr^{0.67} - 1)}\cdot\left(\dfrac{\mu_b}{\mu_s}\right)^n$	Fits the experimental data better; $n = 0.11$ for heating of fluids, $n = 0.25$ for cooling of fluids, $n = 0$ for constant heat flux, $\mu_b/\mu_s = T_s/T_b$, temperature in Kelvin
Flow of liquid metals inside smooth pipes: constant surface heat flux.	$Nu = 4.82 + 0.0185\cdot Pe^{0.827}$	$3600 < Re < 9.05\times10^5$, $100 < Pe < 10{,}000$
Flow of liquid metals inside smooth pipes: constant surface heat flux.	$Nu = 6.3 + (0.0167\cdot Re^{0.85}\cdot Pr^{0.93})$	Recent correlation which fits experimental data better.

Contd.

Contd.

Flow of liquid metals inside smooth pipes: constant surface temperature.	$Nu = 5.0 + 0.025 \cdot Pe^{0.8}$ ...for T_s = constant, $Pe > 100$.	
Helically coiled tubes: Turbulent flow: Hausen's relation	$\frac{Nu_{a\_helical}}{Nu_{a\_straight}} = 1 + \left(\frac{21}{Re^{0.14}}\right) \cdot \left(\frac{D}{d_c}\right)$	D = diameter of tube d_c = diameter of helix

9.12 Summary

Convection is the mode of heat transfer with fluid motion. If the fluid motion is caused by density differences as a result of temperature differences, then it is called 'natural convection'; instead, if the fluid motion is imposed due to a pump or fan, then it is called 'forced convection'. Also, the flow may be 'laminar' or 'turbulent'; in laminar flow, the flow is 'ordered' and the different layers of fluid flow parallel to each other in an orderly manner. In turbulent flow, the flow is 'chaotic' and the flow is highly disordered and there is 'mixing' between different layers of fluid as a result of chunks of fluid ('eddies') moving between layers. Dimensionless number that characterizes the flow as laminar or turbulent is the Reynolds number.

In this chapter, we studied the principles of forced convection and stated a few correlations for external flow on flat plates, cylinders and spheres, and also for internal flow through circular and non-circular pipes.

Mathematical analysis of convection problem is complicated since the temperature profile has to be solved in conjunction with the fluid flow relations. 'Boundary layer concept' simplifies this problem to some extent. Boundary layer is a very thin, stagnant fluid layer that adheres to the wall surface wherein the velocity and temperature gradients are significant. Thus, the flow field is considered to be made up of two regions, one 'a boundary layer region' and the other, an 'inviscid region'. Derivation of boundary layer equations and their solution to the simple case of a flat plate was explained in some detail. Further simplification with the method of integral equations was also demonstrated.

Central problem in convection heat transfer situation is to find out the heat transfer coefficient, 'h'. Heat transfer coefficient is generally represented in terms of the dimensionless Nusselt number, Nu. So, in the analysis, our aim is to get a relation for Nusselt number. By 'Dimensional Analysis', it was shown that in forced convection, Nusselt number is expressed as function of Reynolds and Prandtl numbers.

We are also interested in the drag force between the fluid and the plate and the pressure drop occurs in the pipe if a fluid is flowing through it. This is related to the shear stress at the walls, which in turn, is expressed in terms of a 'skin friction coefficient' for the flat plate and a 'friction factor' for internal flow through a pipe. We solve the momentum equation to get the shear stress and the friction coefficient, and by solving the energy equation we get the temperature profile and thus the heat transfer coefficient.

There is a similarity in the governing equations of momentum and energy transfer. This leads to the idea of 'analogy between momentum and heat transfer' and we have extremely useful analogies such as Reynolds analogy and Colburn analogy. Particularly for rough tubes, an estimate of heat transfer coefficient is easily made just by the knowledge of friction coefficient, with the help of these analogies.

Most of the convection correlations are empirical, deduced as result of large amount of experimental data. Several empirical correlations for laminar as well as turbulent, forced convection, for many practically important situations have been presented in this chapter.

In the next chapter, we study about heat transfer with natural convection.

Questions

1. Explain the difference between natural and forced convection in laminar and turbulent flow. [M.U.]
2. Write short notes on hydrodynamic and thermal boundary layers. What is the importance of these boundary layers in heat transfer? [M.U.]
3. Explain the principle of dimensional analysis. What are its advantages and limitations? [M.U.]
4. State Buckingham π-theorem. [M.U.]
5. Using dimensional analysis, derive an expression for heat transfer coefficient in forced convection in terms of Nusselt number, Reynolds number and Prandtl numbers. [M.U.]

6. Explain the physical significance of: (a) Reynolds number (b) Prandtl number, and (c) Nusselt number [M.U.]
7. Write a short note on applications of dimensional analysis. [M.U.]
8. Write a short note on Reynolds analogy between momentum and heat transfer, with reference to a flat plate. [M.U.]
9. In flow across a cylinder, what is meant by friction drag and pressure drag? At what point on the cylinder is the heat transfer maximum?
10. Show that for flow inside a circular tube, Reynolds number can be written as: $Re = 4.m/(\pi.D.\mu)$
11. State the generally accepted values of critical Reynolds numbers at which the flow changes from laminar to turbulent for: (a) a flat plate (b) flow across a circular cylinder (c) flow across a sphere, and (d) flow inside a circular pipe.
12. Comment on the hydrodynamic and thermal entry lengths for laminar and turbulent flows for an oil inside a circular pipe. How would they compare for a liquid metal?
13. What is the difference between friction factor and friction coefficient?
14. How is pressure drop in a tube related to the friction factor?
15. In a circular tube, where is the heat transfer coefficient higher, at the entry or exit? Why?
16. Does the roughness of tube surface affect the heat transfer in (a) laminar flow (b) turbulent flow. Explain your answer.

Problems

1. Glycerine at 10°C flows over a flat plate, 6 m long, maintained at 30°C with a velocity of 1.5 m/s. Determine the total drag force and the heat transfer rate over the entire plate per unit width. Properties of glycerine at 20°C are: $\rho = 1264$ kg/m^3, $\nu = 1180 \times 10^{-6}$ m^2/s, $Pr = 12{,}500$, $k = 0.2861$ W/ (mK) and $C_p = 2387$ J/(kgK).
2. Water at at 30°C is flowing with a velocity of 4 m/s along the length of a long, flat plate, 0.3 m wide, maintained at 10°C.
   (a) Calculate the following quantities at $x = 0.3$ m:
   (i) boundary layer thickness (ii) local friction coefficient (iii) average friction coefficient (iv) local shear stress due to friction (v) thickness of thermal boundary layer (vi) local convection heat transfer coefficient (vii) average heat transfer coefficient (viii) rate of heat transfer from the plate between $x = 0$ and $x = x$, by convection, and (ix) total drag force on the plate between $x = 0$ and $x = 0.3$ m
   (b) Also, find out the value of x_c (i.e. the distance along the length at which the flow turns turbulent, $Re_c = 5 \times 10^5$).

   Properties of water at a film temperature of 20°C are: $\rho = 1000$ kg/m^3,
   $\nu = 1.006 \times 10^{-6}$ m^2/s, $Pr = 7.02$, $k = 0.5978$ W/ (mK) and $C_p = 4178$ J/(kgK).
3. Consider water flowing at 30°C over a flat plate 1 m x 1m size, maintained at 10°C with a free stream velocity of 0.5 m/s. Plot the variation of local heat transfer coefficient along the length if heating starts from 0.25 m from the leading edge.
4. Air at a pressure of 3 atm. and 200°C flows over a flat plate (1 m long × 0.3 m wide), at a velocity of 7 m/s. If the plate is maintained at 40°C, find out the rate of heat removed continuously from the plate. [**Hint:** heat is removed from both the surfaces of the plate. Properties k, μ, Pr do not vary much with pressure, but, ρ varies as per the Ideal gas law, viz. $\rho = p/(R.T)$, temperature in Kelvin.]

   Properties of air at 1 atm. and a film temperature of 120°C are: $\rho = 0.898$ kg/m^3,
   $\nu = 25.45 \times 10^{-6}$ m^2/s, $Pr = 0.686$, $k = 0.03338$ W/ (mK) and $C_p = 1009$ J/(kgK).
5. In problem 4, apply the Colburn analogy to estimate the drag force exerted on the plate.
6. Dry air at at atmospheric pressure and 30°C is flowing with a velocity of 2 m/s along the length of a flat plate, (size: 1 m × 0.5 m), maintained at 90°C.

   Using Blasius exact solution, calculate the the heat transfer rate from:
   (a) the first half of the plate (b) full plate, and (c) next half of plate.
7. Air at 25°C and atmospheric pressure is flowing with a velocity of 2.5 m/s along the length of a flat plate, maintained at 55°C. Calculate:

   (i) hydrodynamic boundary layer thickness at 20 cm and 40 cm from the leading edge by the approximate method (ii) mass entrainment rate between these two sections assuming a cubic velocity profile, and (iii) heat transferred from the first 40 cm of the plate.
8. An air stream at 20°C and atmospheric pressure, flows with a velocity of 4 m/s over an electrically heated flat plate (size: 0.6 m × 0.6 m), heater power being 1 kW. Calculate:

(i) the average temperature difference along the plate (ii) heat transfer coefficient, and (iii) temperature of the plate at the trailing edge

9. Sodium–Potassium alloy (25% + 75 %), at 250°C , flows with a velocity of 0.5 m/s over a flat plate (size: 0.3 m × 0.1 m), maintained at 550°C. Calculate:
(i) the hydrodynamic and thermal boundary layer thicknesses (ii) local and average value of friction coefficient (iii) heat transfer coefficient, and (iv) total heat transfer rate
Properties of Na–k alloy at a film temerature of 400°C are: ρ = 775 kg/m3, ν = 0.308 × 10^{-6} m^2/s, Pr = 0.0108, k = 22.1 W/ (mK) and C_p = 1000.6 J/(kgK).

10. A thin plate of length 2 m and width 1.5 m is exposed to a flow of air parallel to its surface along the 2 m side. The velocity and temperature of the free stream flow of air are 3 m/s and 20°C respectively. The plate surface temperature is 90°C. Determine the lengthwise mean local heat transfer coefficient at the end of the plate and the amount of heat transferred. Take the following properties of air at 20°C:
ν = 15.06 × 10^{-6} m^2/s, Pr = 0.703, k = 0.0259 W/ (mK), and use the relation: [M.U.]
$Nu_{avg} = 0.664.\ Re_L^{0.5}.\ Pr^{1/3}$.

11. A 10 cm diameter steam pipe, whose surface is at 90°C passes through an area where the wind is blowing across the pipe at a velocity of 40 km/h at a temperature of 10°C and pressure of 1 atm. Determine the rate of heat loss from the pipe per unit length.

12. A 6 mm diameter electrical cable carries a current of 60 Amp and its resistance is 0.002 ohm/metre. Determine the surface temperature of the cable if air at a temperature of 10°C blows across the cable with a velocity of 50 km/h.

13. An incandescent bulb (60 W) can be considered as a sphere of 10 cm diameter. Only 10% of the energy supplied is converted to light and the remaining 90% of the energy is converted to heat. If air at 20°C blows across the bulb with a velocity of 2.5 m/s, determine the equilibrium temperature of the glass bulb.

14. A sphere suspended in an air stream is used as speed measuring device. A 12 mm diameter sphere, when suspended in an air stream flowing at 40°C, maintains a surface temperature of 50°C, while dissipating an electrical energy of 0.6 W. Calculate the air speed.

15. A 1.5 cm diameter ball bearing at a temperature of 100°C is cooled by passing water at a temperature of 15°C at 0.3 m/s over it. Calculate the value of average surface heat transfer coefficient between the ball bearing and water.

16. In a packed bed heat exchanger, air is heated from 30°C to 370°C by passing it through a 10 cm diameter pipe, packed with spheres of 6 mm diameter. The flow rate is 18 kg/h. Pipe surface temperature is maintained at 420°C. Determine the length of bed required. (**Hint:** In this case, heat is transferred between the gas and the walls of the bed).

17. In a regenerator, (1 m dia × 2 m long), spherical rock fillings of diameter = 25 mm, are used to heat up air. Void fraction of this bed is 40%. Initially, the rock fillings are at 25°C and the air is at 85°C, flowing in the axial direction with a flow rate of 1.2 kg/s. Calculate the value of heat transfer coefficient (**Hint:** In this case, heat is transferred between the gas and the spherical fillings).

18. A tube 15 mm ID is maintained at a constant temperature of 60°C. Water is flowing inside the tube at a rate of 10 g/s. Temperature of water at entry is 20°C and at a distance of 1 m from entry the temperature is 40°C. Compute the average value of Nusselt number using the appropriate correlations.

19. Water at 20°C flows through a 15 mm ID, 4 m long tube with a velocity of 2 m/s. Tube wall is maintained at a constant temperature of 90°C. What is the heat transfer coefficient and the total amount of heat transferred, if the exit temperature of water is 60°C? Also, calculate the pressure drop.

20. If, in problem 15, three Globe valves are introduced in the pipe line, what will be the new pressure drop value?

21. Water at 20°C flows through a 15 mm ID, 4 m long tube with a velocity of 2 m/s. Tube wall is maintained at a constant heat flux by electrical heating. What is the heat transfer coefficient and the total amount of heat transferred, and the temperature of tube wall at the exit, if the exit temperature of water is 60°C?

22. In a heat exchanger, water flows through a long 2.2 cm ID copper tube at a bulk velocity of 2 m/s and is heated by steam condensing at 150°C on the outside of the tube. The water enters at 15°C and leaves at 60°C. Find the heat transfer coefficient for water. Use the empirical relation: $Nu = 0.023.Re^{0.8}.Pr^{0.4}$. Physical properties of water at the mean bulk temperature of 37.5°C are: ρ = 990 kg/m^3, μ = 0.00069 kg/(ms), Pr = 0.0108, k = 0.63 W/(mK) and C_p = 4160 J/(kgK). [M.U.]

23. A water heater consists of a thick walled tube of 20 mm ID and 40 mm OD, insulated on the outside surface. Electrical heating within the wall provides a uniform heat generation rate of 5 × 10^5 W/m^3. Water at a rate of 0.15 kg/s enters at 20°C and leaves at 70°C. Calculate the length of tube required. What is the local heat transfer coefficient at the exit, if the inner wall surface temperature at exit is 80°C?

24. Water is flowing through a tube of 6 mm ID at a rate of 4 kg/s. A constant heat flux of 250 W per metre length is provided at the surface. If the water enters at 20°C and exits at 70°C, what is the length of tube required? Also, what is the surface temperature at exit?

25. Liquid sodium is to be heated from 170°C to 230°C at a rate of 2 kg/s in a 2.5 cm diameter tube, heated electrically on its surface. (Constant heat flux). Calculate the length of tube required if the wall temperature is not to exceed 280°C. Properties of sodium at average bulk temperature of 200°C are: $\rho = 903$ kg/m^3, $\nu = 0.506 \times 10^{-6}$ m^2/s, $Pr = 0.0075$, $k = 81.41$ W/(mK) and $C_p = 1327.2$ J/(kgK).

26. A square duct of 20 cm side carries cool air at 10°C over a length of 25 m. Average velocity at entrance is 1.5 m/s. If the walls of duct are maintained at 30°C, determine the outlet temperature of air.

27. Water flows through a rough pipe of 40 mm ID and 3 m length. Relative roughness, (ε/D) for pipe = 0.004. Inlet temperature of water is 20°C and the inlet flow velocity is 1.5 m/s. Determine the outlet temperature and also the pressure drop.

28. Consider a tube bank, made of tubes of 10 mm OD, in an in-line arrangement, longitudinal spacing and transverse spacing being 15 mm and 17 mm respectively. Air is heated from 20°C to 40°C by pumping it through this tube bank. Air approaches the tube bank with a velocity of 4 m/s, and the tube walls are maintained at a constant temperature of 150°C. If there are 10 tube rows, what is the average heat transfer coefficient and the pressure drop?

29. Water at 20°C flows across a tube bundle at a free stream velocity of 20 m/s. OD of the tubes is 8 cm. Longitudinal and transverse spacings are 22.5 cm each. Tubes are in a staggered arrangement. If the tube surfaces are maintained at 50°C, estimate the heat transfer coefficient.

30. Engine oil is to be cooled from 150°C to 90°C in an annulus of 15 mm ID and 30 mm OD. Flow velocity is 1 m/s. Temperature of inside tube wall is maintained at 25°C. Determine the heat transfer coefficient and the length of tube required. Properties of engine oil at a mean bulk temperature of 120°C are: $\rho = 828$ kg/m^3, $\nu = 12 \times 10^{-6}$ m^2/s, $Pr = 175$, $k = 0.1349$ W/ (mK) and $C_p = 2307$ J/(kgK).

31. Water is flowing at the rate of 20 kg/min. through a tube of inner diameter 2.5 cm. The surface of the tube is maintained at 100°C. If the temperature of water increases from 25°C to 55°C, find the length of tube required. Following empirical relation can be used:

$Nu = 0.023.Re^{0.8}.Pr^{0.4}$. Physical properties of water can be taken from the following table: [M.U.]

t, (deg. C)	ρ, (kg/m^3)	C_p, (J/(kgK))	$K \times 10^2$, (W/(mK))	$\mu \times 10^6$, (kg/(ms))
40	992.2	4174	63.35	652
50	988.1	4178	64.74	550
60	983.2	4182	65.90	470
70	977.8	4187	66.72	405
80	971.8	4195	67.41	355